CHAPTER

2

TI-83 Plus, TI-83 Plus Silver Edition, TI-84 Plus, TI-84 Plus Silver Edition, and TI-89 Titanium Graphing Calculators

These three graphing calculators are versatile tools for exploring mathematics. In addition to all of the features of a scientific calculator, they have large-screen computation and programming capabilities and built-in software for working with graphs, tables, lists, matrices, sequences, probability, and statistics. Hence, these calculators are actually powerful, user-friendly hand-held computers.

This chapter is designed to familiarize you with many aspects of these calculators. The models are similar so in most cases you can follow the same instructions, and we will refer to "your calculator," rather than the particular model number. When they do differ, specific instructions will be given for the TI-83 and TI-84, and also for the TI-89. Also, unless otherwise noted, both the TI-83 Plus and the TI-83 Plus Silver Edition will be referred to as the TI-83 and the TI-84 Plus and the TI-84 Plus Silver Edition will be referred to as the TI-84.

Have the calculator out and "on" so that you can work through the examples as you read this chapter. Feel free to explore the menus and features of your calculator. A few hours of productive play can help you reach a comfort level so that you can readily solve problems using this powerful tool.

2.1 Getting Started

2.1.1 Exploring the Keyboard

Take a minute to study the keys on your calculator. There are 10 rows of keys, each with five keys, except for the four specially arranged cursor-movement keys. These keys are divided into three zones.

- **Row 1**
  Used for graphing and table building.
- **Rows 2, 3, and 4**
  Used for accessing menus and editing.
- **Rows 5–10**
  Used like those on a scientific calculator.

Thinking in terms of these three zones will help you find keys on your calculator.

2.1.2 Using the Multipurpose [ON] Key

The On key [ON] is in the lower left-hand corner of the keyboard. It is used to do the following:

- Turn on the calculator.
- Interrupt graphing if you want to stop before a graph is completely drawn.
- Interrupt program execution to break out of a program.
- Turn off the calculator. To do this, press

[2nd] [ON].

Note that the word OFF is written in colored letters just above [ON] and that the color of the letters matches that of [2nd]. In the future, we say, "press [2nd] [OFF]."

To prolong the life of the batteries, your calculator automatically turns itself off after several minutes have elapsed without any activity. To turn on your calculator in these circumstances, press

[ON].

Your calculator will turn on and return you to the screen on which you were working when it turned itself off.

2.1.3 Adjusting the Screen Contrast

You can adjust the screen contrast as needed, choosing from 10 contrast settings that range from 0 (the lightest) to 9 (the darkest).

To darken the screen,

1. press and release [2nd] and then
2. press and hold [▲].

To lighten the screen,

1. press and release [2nd] and then
2. press and hold [▼].

If you find it necessary to set the contrast at 8 or 9, it is probably time to change your batteries. (Your calculator uses four AAA batteries.) If after you change the batteries the screen is too dark, simply adjust contrast following the steps outlined above.

2.2 Calculating and Editing

2.2.1 Returning to the Home Screen

Computation is done on the Home screen. To help you remember how to get to the Home screen from other screens and menus, remember the sentence, "Quit to go Home." This means that if you get lost in a menu and want to return to the Home screen, press

[2nd] [QUIT].

On the TI-89, this key sequence will bring you to the main menu. To enter the home screen, press [HOME].([QUIT] is the second function of [MODE] located to the right of [2nd].) If your calculator does not respond to this command, it is probably busy graphing or running a program. In this case, press

[ON] and then [2nd] [QUIT].

2.2.2 Performing Simple Calculations

1. To compute $2 + 5 \times 8$, press:

2 [+] **5** [×] **8** [ENTER].

Your screen should look like Figure 2.1.

2. Find the value of log(100) by pressing
   - on the TI-83 and TI-84 [LOG] **100** [)] [ENTER], or
   - on the TI-89 [CATALOG] [ALPHA] [ALPHA] [L]. Scroll down to [log (] and press [ENTER] **100** [)] [ENTER].

Note that on the TI-83 the left parenthesis automatically appears after pressing [LOG]*. Your screen should look like Figure 2.2.*

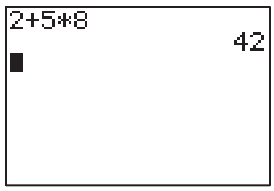


Figure 2.1

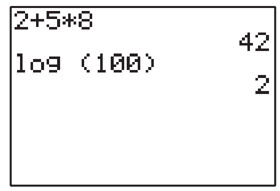


Figure 2.2

Note: *Do not type the letters L, O, and G. The calculator would interpret this as implied multiplication of the variables L, O, and G.*

2.2.3 Working with Error Messages

Your calculator knows the difference between the binary operation of subtraction (the blue [−]) and the additive inverse, or "sign change," operation (the gray or white [(−)]). To learn how the calculator handles errors related to these keys, let's purposely make a mistake. Enter the following key sequence:

7 [+] [−] **4** [ENTER].

Your calculator should respond as shown in Figure 2.3. In this case the *error message* indicates you have made a syntax error and have two choices. This ERROR MESSAGE menu is typical of all numbered menus on your calculator. To select an item from a numbered menu, do either of the following:

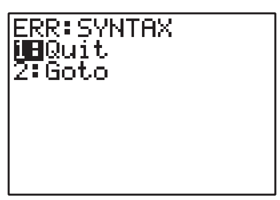


Figure 2.3 The ERROR MESSAGE menu on the TI-83.

a. press the number to the left of the choice you want—this is the fastest way—or

b. position the cursor next to your choice and press [ENTER].

To return to the Home screen (Remember, "Quit to go Home."), press

[2nd] [QUIT],

or press the number that corresponds to [QUIT] on your calculator. Choose Quit.

The screen should look like Figure 2.4, with a flashing cursor below the 7.

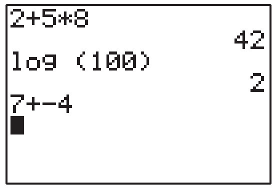


Figure 2.4

To return to the ERROR MESSAGE menu (see Fig. 2.3), press [ENTER].

Selecting the Goto option at this point causes the cursor to go to the source of the error and clears the Home screen of all data except the expression that contains the error. Generally, the Goto option will help you find your error.

1. If you have not already done so, choose the Goto option now.

The cursor should flash on the subtraction symbol.

2. Press [(−)] to overwrite the subtraction symbol with a negative sign.
3. Press [ENTER] to re-execute the calculation.

You should obtain the expected result: 3.

2.2.4 Editing Expressions

Using Last Entry. When you press [ENTER] on the Home screen to evaluate an expression or execute an instruction, the expression or instruction is stored with other previous entries in a storage area called the Last Entry Stack. You can recall a prior entry from the Last Entry Stack, edit it, and then execute the edited instruction, as the following example illustrates.

Example 1 Doubling an Investment's Value

Problem You deposit $500 in a savings account with a 4.75% annual percentage rate (APR), compounded monthly. How long will it take for your investment to double in value?

Solution Because $4.75 \approx 5$ and $100 \div 5 = 20$, you might make an initial guess of 20 years. To check the guess, do the following:

1. Press [2nd] [QUIT] to return to the Home screen, if necessary.
2. Press [CLEAR] once or twice.

   On a line with text on the Home screen, [CLEAR] clears the text from the line.

   On a blank line on the Home screen, [CLEAR] clears the text from the entire screen.
3. Press **500** [(] **1** [+] **0.0475** [÷] **12** [)] [^] [(] **12** [×] **20** [)] [ENTER].

   (See Fig. 2.5.)

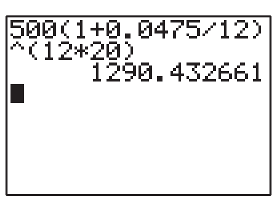


Figure 2.5

4. To display the results in a format more appropriate for calculations involving money,
   a. Press [MODE] to display the MODE screen.
   b. For the TI-83 and TI-84 press [▼] [▶] [▶] [▶] to position the cursor over the 2. For the TI-89 Titanium press [▼][▼][▶] [ALPHA] [G].
   c. Press [ENTER].

The numerical display format is changed to two fixed decimal places (see Fig. 2.6).

(a) (b)

Figure 2.6 The Mode screen on the (a) TI-83 and (b) TI-89 Titanium.

5. Press [2nd] [QUIT] to return to the Home screen.

6. Press [ENTER] to display the result in the two-decimal-place format (see Fig. 2.7).

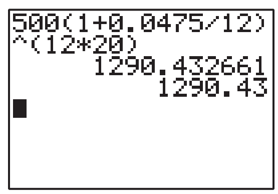


Figure 2.7

Our next guess should be quite a bit less than 20 years, say 14 years. In this case, do the following:

1. To edit the old expression, press [2nd] [ENTRY] [◀] [◀] [◀] **14.**

Grapher Note: *The first two keys of this sequence are not needed for the TI-89 Titanium.*

2. Evaluate the edited version by pressing [ENTER] (see Fig. 2.8).

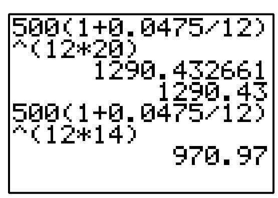


Figure 2.8

3. To change the number of years to 14.5, press

[2nd] [ENTRY] [◀] [•] **5** [ENTER].

Notice that the final parenthesis can be left off and that all three results can be seen on the screen (see Fig. 2.9).

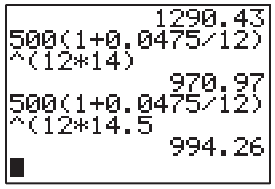


Figure 2.9

Continue this guess-and-check procedure until you obtain the accuracy you desire. Press [2nd] [ENTRY] several times to observe how the Last Entry Stack has stored several prior entries.

Display Cursors. There are four types of display cursors. Each of these cursors indicates what will happen when you press the next key (see Table 2.1).

Table 2.1 Display cursors.

Entry cursor	Solid blinking rectangle	The next keystroke is entered at the cursor; it overwrites any character.
INS (insert) cursor	Blinking underline	The next keystroke is inserted in front of the cursor location.
2nd cursor	Blinking ↑	The next keystroke is a 2nd operation.
ALPHA cursor	Blinking A	The next keystroke is an alphabetic character. The SOLVE command may be executed on the TI-83.

Using the Edit Keys. The Edit keys help you make effective use of your calculator. Study Table 2.2.

Table 2.2 Edit keys.

Key	**Comments**
[◀] or [▶]	Moves the cursor within a line. These keys repeat.
[▲] or [▼]	Moves the cursor between the lines. These keys repeat.
[2nd] [◀]	Moves the cursor to the beginning of the expression. Can be used for fast-tracing on the Graph screen.
[2nd] [▶]	Moves the cursor to the end of the expression. Can be used for fast-tracing on the Graph screen.
[ENTER]	Evaluates an expression or executes an instruction. This key acts as a Pause key when graphing, press it a second time to resume graphing.
[CLEAR]	• On a line with text on the Home screen, this key clears (blanks) the current command line. • On a blank line on the Home screen, it clears the screen. • In an editor, it clears (blanks) the expression or value on which the cursor is located. It does not store zero as the value.
[DEL]	Deletes the character at the cursor. This key repeats.
[2nd] [INS]	Inserts characters at the underline cursor. To end the insertion, press [2nd] [INS] or a cursor-movement key.
[2nd]	Means the next key pressed is a 2nd operation (the color-coded operation to the left above a key). The cursor changes to an ↑. To cancel 2nd, press [2nd] again.

[ALPHA]	Means the next key pressed is an ALPHA character (the color-coded character to the right above a key). The cursor changes to an A. To cancel ALPHA, press [ALPHA] or a cursor-movement key.
[◆]	This key has a function similar to [ALPHA], but can be found only on the TI-89 Titanium.
[2nd] [A-LOCK]	Sets ALPHA-LOCK. Each subsequent key press is an ALPHA character. The cursor changes to an A. To cancel ALPHA-LOCK, press [ALPHA]. Note that prompts for names automatically set the keyboard in ALPHA-LOCK.
[X,T,θ,n]	Allows you to enter an X in Function (Func) mode, a T in Parametric (Par) mode, a θ in Polar (Pol) mode, or an n in Sequence (Seq) mode without pressing [ALPHA] first. The TI-89 Titanium does not have a single key for variables. Each variable can be found individually.

2.2.5 Scientific Notation and the Answer Key

Example 2 illustrates a geometric progression—a sequence of numbers that grows by a constant factor—while demonstrating some important features of your calculator.

Example 2 Generating a Geometric Sequence

Problem Display the first few terms of the sequence that begins with 1.7×10^3 and grows by a factor of 100.

Solution To generate the sequence, do the following:

1. Return your calculator to Floating Point Numerical Display (Float) mode by pressing
   - [MODE] [▼] [ENTER] on the TI-83 and TI-84, or
   - [MODE] [▼] [▼] [▶], then scroll to a [Float] number and press [ENTER] [ENTER].
2. Press [2nd] [QUIT] to return to the Home screen.
3. Clear the Home screen by pressing [CLEAR] [CLEAR].
4. To enter 1.7×10^3 onto the Home screen, press **1.7** [2nd] [EE] **3** [ENTER].
   Grapher Note: *Many functions on the TI-89 Titanium can be found only under* [CATALOG]. *This function is under* [CATALOG] *as* [E].

Notice that entering the number in scientific notation did not cause the result to be displayed in scientific notation (see Fig. 2.10).

5. Press [×] **100.**

As soon as you press [×], 'Ans ∗' is displayed on the screen. **Ans** is a variable that contains the last calculated result (see Fig. 2.11).

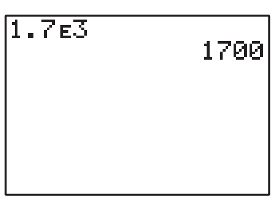


Figure 2.10

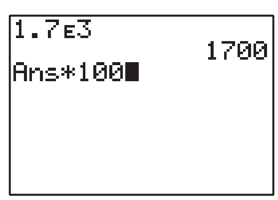


Figure 2.11

6. Press [ENTER] four times.

Each time you press [ENTER]*, the previous answer is multiplied by 100 and Ans is updated. Notice that the displayed values automatically change to scientific notation after the third iteration (see Fig. 2.12).*

7. Press [ENTER] twice to see the geometric progression continue (see Fig. 2.13).

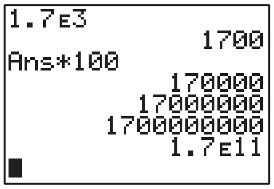


Figure 2.12

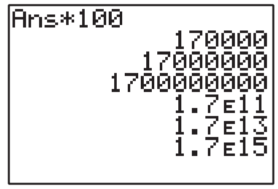


Figure 2.13

2.2.6 Other Computation Features and Menus

Clear the Home screen and then try the following calculations.
Grapher Note: *On the TI-89 press* [ALPHA] [≈] *to find a decimal approximation.*

1. **Integer Arithmetic**

   To calculate $-2 - (-3) + (-4) \times 5$, press

   [(–)] **2** [–] [(–)] **3** [+] [(–)] **4** [×] **5** [ENTER].

2. **Rational-number arithmetic**

   To add the fractions $\frac{1}{3}$ and $\frac{4}{7}$, press

   1 [÷] **3** [+] **4** [÷] **7** [MATH] [1: Frac] [ENTER].

3. **Real-number arithmetic**

   To approximate the principal square root of 10, press [2nd] [√‾] **10** [)] [ENTER].

   (See Fig. 2.14.)

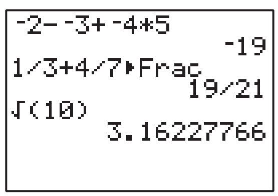


Figure 2.14

4. **Order of operations**

   To show that exponents take precedence over negation, and thus $(-6)^4 \neq -6^4$, press

   [CLEAR] [(] [(–)] **6** [)] [^] **4** [ENTER].

   Then press

   [(–)] **6** [^] **4** [ENTER].

   and compare the results (see Fig. 2.15).

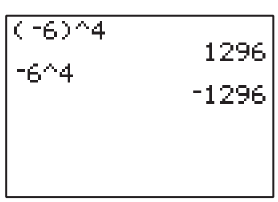


Figure 2.15

5. **Trig and angle computation**

   To calculate tan 60° without switching to Degree mode, press

   - [CLEAR] [TAN] **60** [2nd] [ANGLE] [1: °] [ENTER] on the TI-83 and TI-84, or
   - [2nd] [TAN] **60** [2nd] [°] [◆] [≈] on the TI-89 Titanium.

   Then press

   [2nd] [√‾] 3 [ENTER]

   and compare the results. Re-enter these expressions adding parentheses as needed to match Figure 2.16.

6. **Roots**

   To evaluate $\sqrt[5]{-16807}$, press either

   [CLEAR] **5** [MATH] [5 : $\sqrt[x]{\ }$] [(−)] **16807** [ENTER],

   or

   [( ] [(−)] **16807** [ )] [^] [( ] **1** [÷] **5** [ )] [ENTER].

   (See Fig. 2.17). ***Grapher Note:*** *The TI-89 Titanium requires the use of the second method.*

7. **Greatest integer function**

   To determine the greatest integer less than or equal to −4.916, press [MATH] [▶] [5 : int] [(−)] **4.916** [ENTER].

Add parentheses if you wish to match Figure 2.18. ***Grapher Note:*** [int] *is found under* [CATALOG] *on the TI-89 Titanium.*

8. **Factorial**

   To evaluate 10! = 10 · 9 · 8 · 7 · 6 · 5 · 4 · 3 · 2 · 1, press

   10 [MATH] [◀] [4 : !] [ENTER].

   (See Fig. 2.18) ***Grapher Note:*** [!] *is found under* [CATALOG] *on the TI-89 Titanium.*

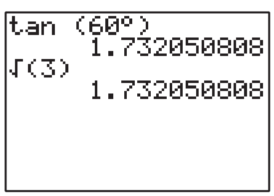


Figure 2.16

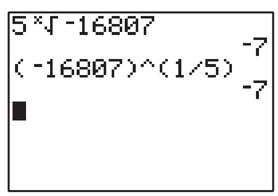


Figure 2.17

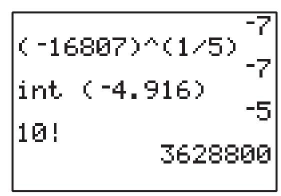


Figure 2.18

2.2.7 Computing with Lists

Set the display format to five fixed decimal places as follows:

1. Press [MODE].
2. On the TI-83 and TI-84 press [▼] and then [▶] six times. On the TI-89 Titanium, press [▼] [▼] [▶] [ALPHA] [J].
3. Press [ENTER].
4. Return to the Home screen by pressing [2nd] [QUIT] .
5. Clear the Home Screen by pressing [CLEAR].

Patterns in logarithmic outputs

Refer to Figure 2.19 as you proceed through these steps:

1. To enter $\log(2^1)$, press

   [LOG] **2** [^] **1** [)] [ENTER].

2. To enter $\log(2^2)$, press

   [2nd] [ENTRY] [◀] [◀] **2** [ENTER].

3. To enter $\log(2^3)$, press

   [2nd] [ENTRY] [◀] [◀] **3** [ENTER].

See Figure 2.19. Do you see the pattern? A rule of logarithms states that for positive numbers x, $\log(x^n) = n \log(x)$. To see the pattern in a different way.

1. Press [LOG] [2nd] [{] **2** [,] **4** [,] **8** [2nd] [}] [ENTER], *adding parentheses if needed.*
2. Press and hold [▶] to see the third item in the "list." (See Fig. 2.20)

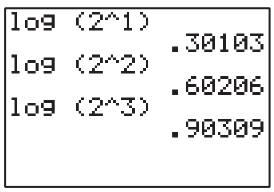


Figure 2.19

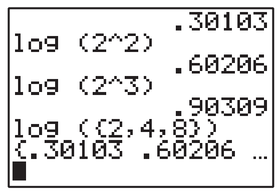


Figure 2.20

The curly braces { } are used to enclose an ordered set of numbers, or **list.** List notation looks just like set notation, but you can add, subtract, multiply, and divide lists, whereas you operate on sets differently, using operations such as union and intersection. Your calculator manual has a chapter on lists. You also can learn about lists through experimentation; try using them in various ways and observe the results.

2.2.8 Using Variables

Example 3 Finding the Height of a Triangle

Problem A triangle encloses an area of 75 cm^2 and has a base of 11 cm. What is its height?

Solution Recall that the area is given by one half the base times the height: $A = (1/2)bh$. Therefore to find the height, do the following:

1. To put your calculator in Floating Point mode,
   a. press [MODE] and
   b. select the Float option.
2. Return to and clear the Home screen.
3. To store the value 11 as the variable B, press

   11 [STO▶] [ALPHA] **B** [ENTER].

4. Because one-half the base is about 5, the height should be about 15. Therefore press

   15 [STO▶] [ALPHA] **H** [ALPHA] [:] [(] **1** [÷] **2** [)] [2nd] [A-LOCK] **B** [×] **H** [ENTER].

   On the TI-89 Titanium, [:] is accessed by pressing [2nd]. (See Fig. 2.21.)

5. Our guess was too big, so enter

[2nd] [ENTRY] [▲] **14** [ENTER].

(See Fig. 2.22.)

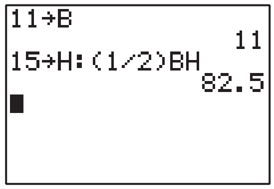


Figure 2.21

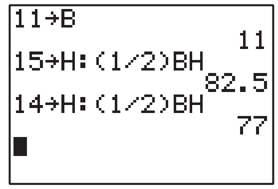
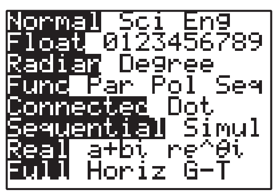


Figure 2.22

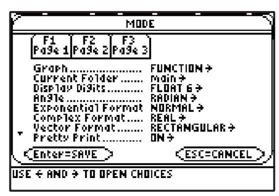
The next guess would be between 13 and 14 and would require inserting extra digits for the number being stored in H (press [2nd] [INS] at the appropriate location). Continue the guess-and-check process to practice using the editing features of your calculator and to find the height with an error of no more than 0.01.

2.3 Function Graphing and Table Building

Graphing and table building on your calculator involve the top row of keys. There are four graphing modes on your calculator: Function, Parametric, Polar, and Sequence. Each has a corresponding table-building mode. Thus changing the setting on the fourth line of the Mode screen affects both graphing and table building (see Fig. 2.23).



(a)



(b)

Figure 2.23 The Mode screen on the (a) TI-83 and TI-84 and (b) TI-89 Titanium.

For this section, be sure your calculator is in Function mode (Func). In Section 2.4 we explore the Parametric and Polar modes. The remainder of this section is built around various calculator methods for solving equations, using the example

$$\cos x = \tan x \text{ for } 0 \le x \le 1.$$

2.3.1 Method A: Graphing Each Side and Zooming In

1. Enter each side of the equation as a function on the Y = screen by pressing
   - [Y=] [COS] [X,T,θ,n] [ENTER] [TAN] [X,T,θ,n] [ENTER] on the TI-83 and TI-84, or
   - [◆] [Y=] [2nd] [COS] [X] [ENTER] [2nd] [TAN] [X] [ENTER] on the TI-89 Titanium.

   Insert parentheses if you wish to match Figure 2.24.

(a) (b)

Figure 2.24 The Y = screen on the (a) TI-83 and TI-84 and (b) TI-89 Titanium.

2. Press [ZOOM] [4 : ZDecimal].

   Watch as the curves are graphed in sequence. The vertical lines are pseudoasymptotes of y = tan x. The calculator is actually connecting points that are off the screen (see Fig. 2.25).

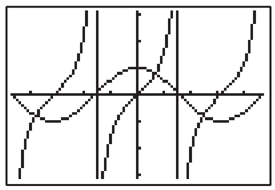


Figure 2.25

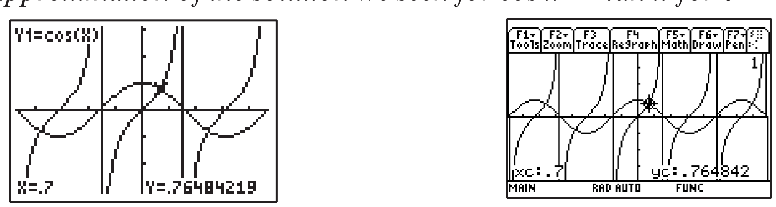
3. Press [WINDOW] to see what portion of the plane is being used for graphing. The viewing rectangle, or window, being used is [Xmin, Xmax] by [Ymin, Ymax], in this case [−4.7, 4.7] by [−3.1, 3.1]. Because Xscl = 1 and Yscl = 1, the tick marks on each axis are one unit apart (see Fig. 2.26). The TI-83 has an extra line on the Window screen to set the resolution. For our purposes, keep Xres = 1.

(a) (b)

Figure 2.26 The Window editor screen on the (a) TI-83 and TI-84 (b) TI-89 Titanium.

4. Press [TRACE]

   Observe the coordinate readout at the bottom of the screen as you press and release [▶] *repeatedly. Stop when x = 0.7. The graphs appear to intersect at x = 0.7; actually this is a rough approximation of the solution we seek for cos x = tan x for 0 ≤ x ≤ 1(see Fig. 2.27).*



(a) (b)

Figure 2.27 Tracing on the (a) TI-83 and TI-84 and (b) TI-89 Titanium.

Now you can probably see why the fourth ZOOM feature is called Zoom Decimal (ZDecimal). It adjusted the viewing window to give a nice *decimal* readout. Notice the 1 in the upper right-hand corner of the TI-82 screen. It lets you know that you are tracing on Y_1, which in this case is cos *x*. The TI-83 shows the equation.

5. Press [▼] to move the Trace cursor to Y_2.

   The x value does not change, but the y value does, because you are now tracing on $Y_2 = \tan x$. Notice the screen indicator has changed to show you are tracing on Y_2 (see Fig. 2.28).

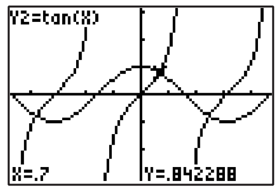


Figure 2.28 TI-83 and TI-84 versions

6. Press [GRAPH].

   The Trace cursor, the coordinate readout, and the number in the upper right-hand corner of the screen all disappear and only the graph itself is displayed (see Fig. 2.29).

7. Press any of the cursor-movement keys. You now are using a free-moving cursor that is not confined to either of the graphs. Notice that this cursor looks different from the Trace cursor.

8. Experiment with all four cursor-movement keys.

   Watch the coordinate readout change. Move to the point (0.7, 0.8). Notice $y = 0.8$ is not the value of either function at $x = 0.7$, it is just the *y*-coordinate of a dot (pixel) on the graphing screen (see Fig. 2.30). The coordinates (0.7, 0.8) are the *screen coordinates* of the pixel. Notice that the free-moving cursor yields a nice decimal readout for both *x* and *y*. This is because we used Zoom Decimal to set the viewing window.

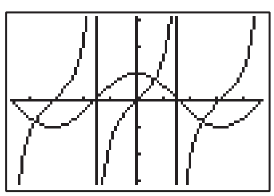


Figure 2.29

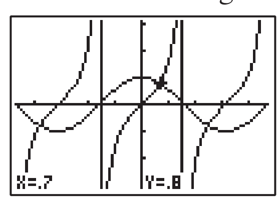


Figure 2.30

Using ZOOM Box. This option lets you use the cursor to select opposite corners of a "box" to define a new viewing window. Continuing the example from above, do the following:

1. Press [ZOOM] [1 : Box]. Then move the cursor to (0,0). (See Fig. 2.31.)

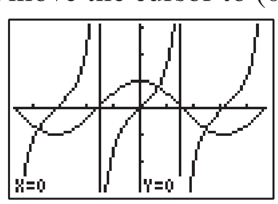


Figure 2.31

2. To select a new viewing window of [0, 1] by [0, 1.2], which will limit x so that

$$0 \le x \le 1,$$

   a. press ENTER to select the point (0, 0) as one corner of the new viewing window and
   b. use the cursor-movement keys to move to the opposite corner (1, 1.2). (See Fig. 2.32.)

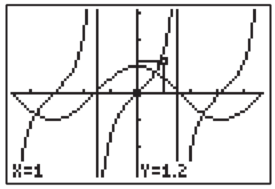


Figure 2.32

3. To select (1, 1.2) as the opposite corner of the new viewing window, press

ENTER.

The graphs of the two functions will be drawn in the new viewing window (see Fig. 2.33).

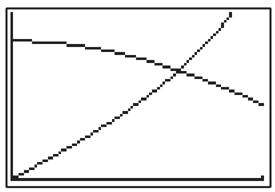


Figure 2.33

4. To remove the cursor and coordinates from the screen, press [GRAPH].

5. To verify that the new viewing rectangle is [0,1] by [0, 1.2], press [WINDOW].

Notice that Xscl and Yscl are still both equal to one. The Zoom Box option does not change the scale settings (see Fig. 2.34).

6. To approximate the solution as $x \approx 0.6702$,
   a. press [TRACE] and
   b. use the cursor-movement keys to move to the point of intersection (see Fig. 2.35).

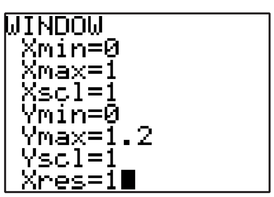


Figure 2.34 TI-83 and TI-84 versions

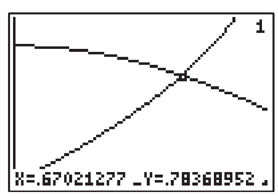


Figure 2.35 TI-83 and TI-84 versions

Finding an error bound. Next, using the approximate solution we found in number 6 above, we want to find the error bound for *x,* as follows:

1. To return to and clear the Home screen, press [2nd] [QUIT] [CLEAR].
2. To see the approximate solution on the TI-83 and TI-84, press [X,T,θ,n] [ENTER]. The TI-89 Titanium stores the value of the intersection as xc, requiring you to press [×] [ALPHA] [C] [ENTER].
3. Press
   - on the TI-83 and TI-84 [VARS] [1 : Window] [8 : ΔX] [ENTER].
   - on the TI-89 [2nd] [CHAR] [1:Greek] [5:Δ] [ENTER].

The value of Δx is the horizontal distance between consecutive pixels in the current viewing window, which in this case is about 0.011. This is an error bound for x. Our approximate solution 0.6702, has an error of at most 0.011.

We need to pick Xmin and Xmax so that they are closer together to decrease this error bound (see Fig. 2.36).

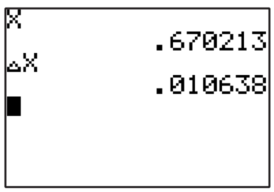


Figure 2.36a TI-83 and TI-84 versions

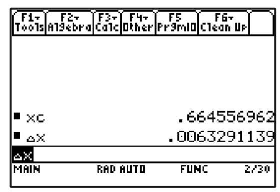


Figure 2.36b TI-89 Titanium version

Do the following:

1. To enter the smaller window of [0.5, 0.8] by [0.6, 1.0], press [WINDOW], followed on the TI-84 by [▼]; then press

   0.5 [ENTER] **0.8** [ENTER] **0.1** [ENTER] **0.6** [ENTER] **1** [ENTER] **0.1** [ENTER].

   (See Fig. 2.37)

2. To move to the point of intersection—approximately (0.666, 0.786), press

   [TRACE]

   and then after the graph is drawn use the cursor-movement keys (see Fig. 2.38).

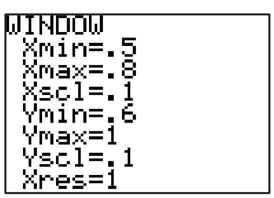


Figure 2.37 TI-83 and TI-84 versions

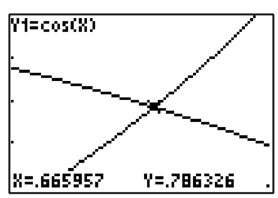


Figure 2.38 TI-83 and TI-84 versions

3. To display the previous approximation and error bound along with the new and improved approximation and error bound (see Fig. 2.39), press
   - on the TI-83 and the TI-84 [2nd] [QUIT] [X,T,θ,n] [ENTER] [VARS] [1 : Window] [8 : ΔX] [ENTER].
   - on the TI-89 Titanium [HOME] [X] [ALPHA] [C] [ENTER] [2nd] [CHAR] [1:Greek] [5:Δ] [ENTER].

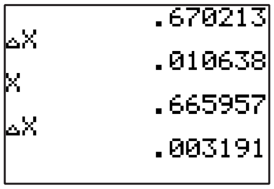


Figure 2.39A TI-83 and TI-84 versions

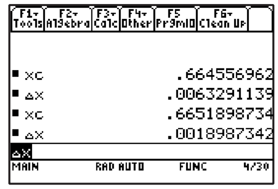


FIGURE 2.39B TI-89 Titanium version

4. Evaluate cos x and tan x on your calculator. You should see the cos x and tan x are nearly, but not exactly, equal when $x = 0.6659\ldots$ (see Fig. 2.40).

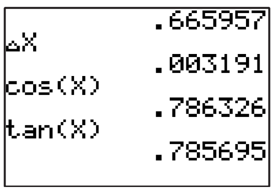


Figure 2.40A TI-83 and TI-84 versions

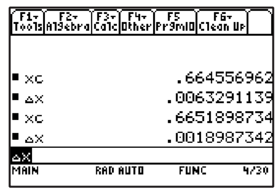
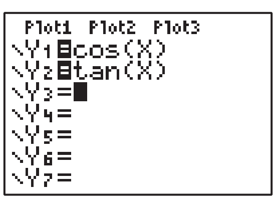


FIGURE 2.40B TI-89 Titanium version

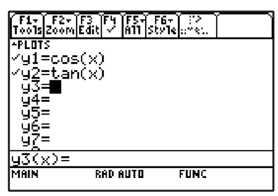
2.3.2 Method B: Table Building

The Y = screen is used to enter functions for both graphing and table building. To build a table, do as follows:

1. Press [Y=] to check that $Y_1 = \cos x$ and $Y_2 = \tan x$ (see Fig. 2.41).



(a)



(b)

Figure 2.41 The Y = screen on the (a) TI-83 and TI-84 and (b) TI-89 Titanium.

2. To reveal the Table Setup screen, press [2nd] [TBLSET].

3. Press **0** [ENTER] **0.1** [ENTER] and ensure the Auto option is selected for both the independent variable (x) and the dependent variable (y) (see Fig. 2.42).

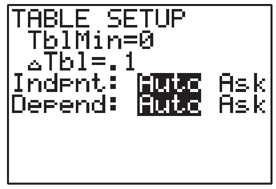


Figure 2.42

4. Press [2nd] [TABLE] and notice that the first x-value is the TblMin ($= 0$) and that the increment from one row to the next in the x column is Δ Tbl ($= 0.1$) (see Fig. 2.43).
5. Press [▼] repeatedly to move down the x column of the table to 0.7. Notice that the solution lies between $x = 0.6$ and $x = 0.7$ (see Fig. 2.44).

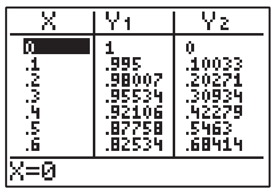


Figure 2.43

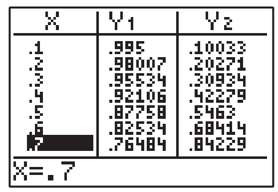


Figure 2.44

Use the cursor-movement keys to move around the table and explore. Pay attention to the readout at the bottom of the screen as you move to different cells in the table.

6. Press

[2nd] [TBLSET] **0.6** [ENTER] **0.01** [ENTER].

The value of Δ Tbl will serve as the error bound for table building, just as Δx did for graphing (see Fig. 2.45).

7. Press [2nd] [TABLE] and then press [▼] repeatedly until you reach $x = 0.67$. This is a solution with an error of at most 0.01 (see Fig. 2.46).

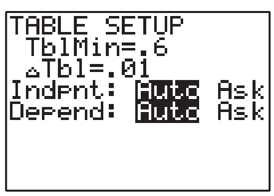


Figure 2.45

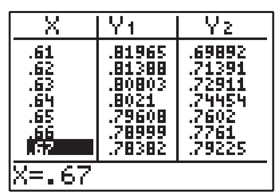


Figure 2.46

2.3.3 Method C: Solving an Equivalent Equation

To solve $\cos x = \tan x$ for $0 \le x \le 1$, you can solve the equivalent equation

$$\cos x - \tan x = 0$$

for the same interval. To do this use the following steps.

1. Press
   - on the TI-83 and TI-84

     [Y=] [▼] [▼] [VARS] [▶] [1: Function . . .] [1:Y_1] [—] [VARS] [▶]
     [1: Function . . .] [2:Y_2] [ENTER],
   - on the TI-89 [◆] [Y=] [▼] [▼] [Y] **1** [(] [X] [)] [—] [Y] **2** [(] [X] [)] [ENTER].

(See Fig. 2.47).

2. To deselect Y_1 and Y_2, press

[▲] [▲] [◀] [ENTER] [▲] [ENTER].

Grapher Note: *There are check marks next to the equations on the TI-89. Pressing* [F4] *will remove these checks and stop the function from being graphed.*

Now only Y_3 should have its equals sign highlighted (see Fig. 2.48).

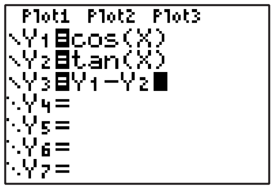


Figure 2.47 TI-83 and TI-84 versions

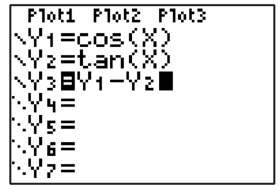


Figure 2.48 TI-83 and TI-84 versions

3. To see the graph of $y = \cos x - \tan x$ in a friendly viewing window, press

[ZOOM] [4 : ZDecimal]; and after the graph is drawn, press

[TRACE] [2nd] [▶] [▶] [▶].

Notice [2nd] [▶] *moves the cursor five pixels to the right for fast tracing (see Fig. 2.49).*

4. To enter the Zoom Factors screen, press

[ZOOM] [▶] [4 : SetFactors…]

and enter 10 as both the horizontal and the vertical magnification factor by pressing

10 [ENTER] **10** [ENTER].

(See Fig 2.50.)

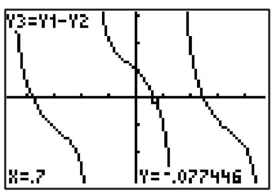


Figure 2.49 TI-83 and TI-84 versions

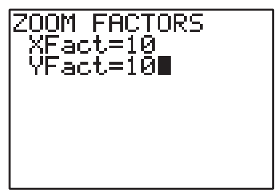


Figure 2.50

5. To center your zoom-in at the point $(x, y) = (0.7, 0)$, press

[ZOOM] [2 : ZoomIn]

and move the cursor to (0.7, 0). (See Fig. 2.51.)

Then press [ENTER] to zoom in.

6. After the graph is redrawn, you can obtain the same approximation that was found by Method B by pressing

[TRACE] [◀] [◀] [◀].

Check the value of Δx; it is the same as the Δ Tbl in method B! (See Fig. 2.52.)

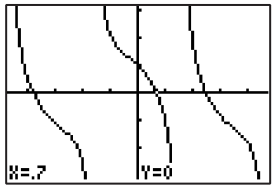


Figure 2.51 TI-83 and TI-84 versions

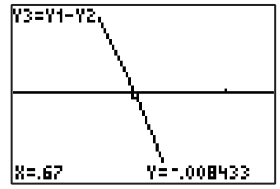


Figure 2.52

2.3.4 Other Equation-Solving Methods

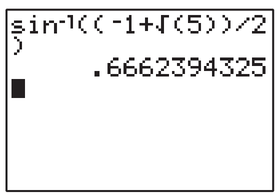
Traditional algebra and trigonometry can be used to determine the exact solution of equation 1.

$$x = \sin^{-1}\frac{-1+\sqrt{5}}{2}$$

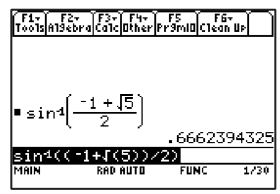
Do the following:

1. To evaluate this expression on your calculator, enter it as shown in Figure 2.53.

You obtain an approximation that is accurate to 10 decimal places. It should be consistent with those found by Methods A, B, and C, and it is (see Fig. 2.53).



(a)



(b)

Figure 2.53 An arcsin computation on the (a) TI-83 and TI-84 and (b) TI-89 Titanium.

2. Set up your Y = screen as you did for Method C. Then, to obtain a graph, press [ZOOM] [4 : ZDecimal].

3. On the TI-83 and TI-84 press [2nd] [CALC] [2 : zero]. On the TI-89 press [F5] [2 : zero].

This should yield a prompt requesting a Lower Bound or Left Bound (see Fig. 2.54).

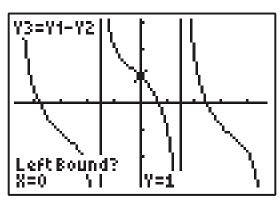


Figure 2.54 Zero finder on the TI-83 and TI-84.

4. Because we are seeking a solution for $0 \leq x \leq 1$, the lower bound should be $x = 0$; so press [ENTER].
5. To move the cursor to $x = 1$, press

[2nd] [▶] [2nd] [▶]

followed by [ENTER] to enter it as the upper bound.
6. Move the Trace cursor to $x = 0.7$ and enter it as your guess by pressing

[◀] [◀] [◀] [ENTER].

The calculator should yield a root value of $x = 0.66623943$ *(see Fig. 2.55).*

7. To compare the value found using the root finder and the value found in Part 1 above, press

- on the TI-83 and TI84, [X,T,θ,n] [ENTER], or
- on the TI-89, [X] [ALPHA] [C] [ENTER].

They match perfectly to 10 decimal places! (See fig. 2.56.)

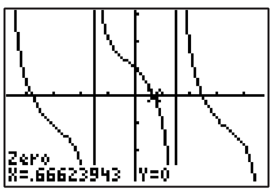


Figure 2.55 TI-83 and TI-84 versions

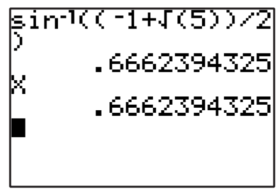


Figure 2.56 TI-83 and TI-84 versions

There are many other ways to solve equations on your calculator. Feel free to explore them.

2.4 Other Graphing and Table Building

2.4.1 Parametric Graphing and Table Building

Parametric equations are ideal tools for representing and solving problems in geometry and the physics of motion. Your calculator has a built-in parametric graphing utility. This utility is similar to the function graphing utility and is almost as easy to use. To graph a parametric curve, you

- select the parametric (Par) mode on the Mode screen,
- type the desired equations in the Y= screen,
- set the intervals for *t, x,* and *y* using the Window screen, and
- press [GRAPH].

Parametric equations are written in the form:

$$x = f(t) \text{ and } y = g(t).$$

In this setting t is called a parameter; however, t actually is an independent variable, not a parameter in the sense that m and b are parameters in the equation $y = mx + b$. Unlike the independent variable x we are used to in Function-graphing mode, the parameter t is not a plotted, visible coordinate; it is hidden from view when we look at a parametric curve. When we use the TRACE feature, we see a readout of the parameter t and the coordinates x and y, which are the dependent variables of the parametric representation.

Example 4 Graphing a Parametric Curve

Problem Graph the curve represented by the following parametric equations:

$$x = t^2 \text{ and } y = t - 1 \text{ for } -2 \le t \le 2.$$

Solution To solve this problem, follow these steps:

1. Press [MODE] to enter the Mode screen and
   a. select Parametric Graphing (Par) and
   b. choose the default (leftmost) settings for the other mode settings.
2. Because we are in Parametric mode, pressing [X,T,θ,n] will yield the letter t. To enter the given parametric equations, press

   [Y=] [X,T,θ,n] [x^2] [ENTER] [X,T,θ,n] [—] **1** [ENTER].

The screen should look like Figure 2.57.

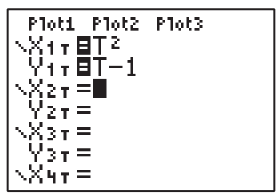


Figure 2.57 The Y= screen on the TI-83 and TI-84.

3. Press [WINDOW] and then set the Window screen as shown in Figure 2.58. (Note that you won't be able to see the entire screen at once because it has too many lines.)

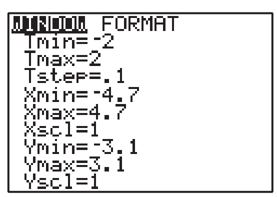


Figure 2.58 Facsimile of the Window screen.

The t step on the Parametric Window screen is the change between the successive t-values that the calculator uses to compute and plot (x, y) pairs. In this case, the t step of 0.1 will yield 40 steps from the t Min of −2 to the t Max of 2. Thus 41 points will be calculated and plotted, with the points corresponding to

$$t = -2.0, -1.9, -1.8, -1.7, \ldots, 1.9, 2.0.$$

Table 2.3 shows the numerical relationship between the parameter t and the coordinates x and y for some of the points to be plotted.

The last two columns of Table 2.3 determine the (x, y) coordinate pairs to be plotted. The values of the parameter t will not appear on the graph.

You can create a table like Table 2.3 on your calculator as follows:

1. Press [2nd] [TBLSET] [(−)] **2** [ENTER] **0.1** [ENTER]. (See Fig. 2.59.)
2. Then press [2nd] [TABLE]. (See Fig. 2.60.)

Table 2.3 Table of Parameter and Coordinate Values

t	$x = t^2$	$y = t - 1$
−2.0	4.00	−3.0
−1.9	3.61	−2.9
−1.8	3.24	−2.8
−1.7	2.89	−2.7
.	.	.
.	.	.
.	.	.
1.9	3.61	0.9
2.0	4.00	1.0

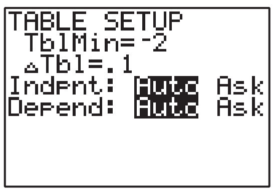


Figure 2.59

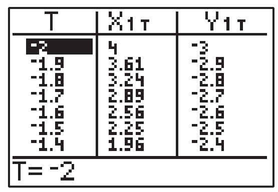


Figure 2.60

To obtain the graph corresponding to Table 2.3 and Figure 2.60, do the following:

1. Press [GRAPH] to yield the plot shown in Figure 2.61.

Because the calculator is in Connected mode, the plotted points in Figure 2.61 are connected by the line segments.

2. To display only the 41 plotted points, choose the Dot mode from the Mode screen and press [GRAPH] again (see Fig. 2.62).

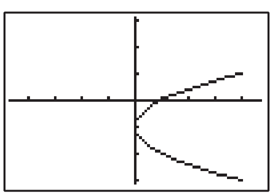


Figure 2.61

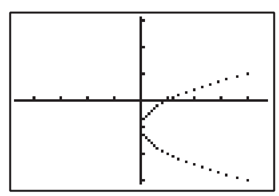


Figure 2.62

Return to Connected mode and use the TRACE feature and the left and right cursor-movement keys to explore the graph numerically. Notice that the values of the parameter t and the x- and y-coordinates are all shown on the screen (see Fig. 2.63 and 2.64). Can you find the six points that correspond to the completed rows of Table 2.3?

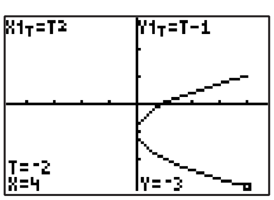


Figure 2.63 TI-83 and TI-84 versions

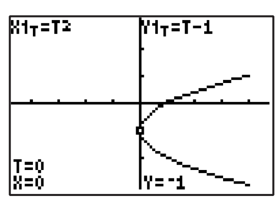


Figure 2.64 TI-83 and TI-84 versions

2.4.2 Polar Equation Graphing

The Polar Equation graphing mode is similar to the other graphing modes.

Example 5 Graphing Two Equations Simultaneously

Problem Graph $r = 9 \sin 5\theta$ and $r = 9$.
Solution

1. Press [MODE] (see Fig. 2.65) and
   a. select Polar (Pol) mode and Simultaneous (Simul) mode and
   b. choose the defaults for the other modes.
2. Enter the Y = Editor to display the Polar Equation screen.
3. To define the two desired equations, press

9 [SIN] **5** [X,T,θ,n] [ENTER] **9** [ENTER].

(See Fig. 2.66.)

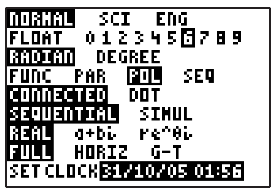


Figure 2.65 TI-83 and TI-84 versions

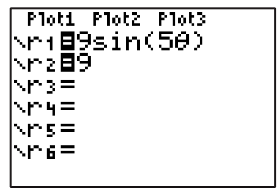


Figure 2.66 TI-83 and TI-84 versions

4. Press [ZOOM] [6 : ZStandard].

The graph of $r = 9$ *is a circle of radius 9 centered at the pole. The circle circumscribes the five-petaled rose curve* $r = 9 \sin 5\theta$ *(see Fig. 2.67).*

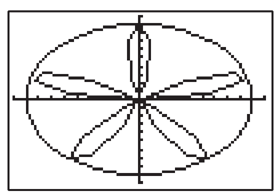


Figure 2.67

5. Set θmax $= \pi$ in the Window screen.
6. To "square up" the window, press

[ZOOM] [5 : ZSquare].

The entire rose curve is plotted using the interval $0 \leq \theta \leq \pi$. *Press* [TRACE] *and explore the two curves (see Fig. 2.68).*

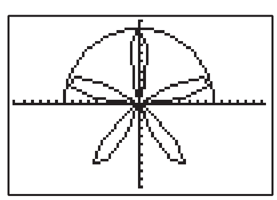


Figure 2.68