

Grapher Workshop

1.1 Introduction

We call this chapter a workshop to emphasize that learning the grapher requires active hands-on work. In thousands of workshop hours over the past 10 years we have helped mathematics students and instructors learn how to use hand-held graphing calculators. Our reward has been the enthusiasm and excitement of the participants as they catch on to the remarkable way that a grapher aids their learning and teaching.

As much as we would like to do it, we cannot bring the actual workshop to you. Nevertheless, with some basic knowledge and your own creativity, you can learn to use and appreciate this technology. A few hours of productive play with this powerful tool will allow you to solve mathematical problems in new ways. As you read the text, you should work through the activities and examples using your grapher. Feel free to explore its menus and features. We suggest that you refer to this workshop whenever you encounter mathematics that require you to use features of the grapher that you are unfamiliar with. The grapher is a powerful tool. With play, thought, and practice, many students have found it both useful and exciting.

1.2 Numerical Computation and Editing

First Steps

Take a moment to study the keyboard of your grapher. The keys are grouped in “zones” according to their function: scientific calculation, graphing, editing, and various menus. Locate **ON**. Not only is it used to turn on your grapher, but it also acts as an **[OFF]** button as its *second function*, **2nd ON**.

- Practice turning your calculator on and off.

Next determine how to adjust the screen contrast, something you may need to do as lighting conditions change or battery power weakens. (You may have to check your grapher owner’s manual to see how this is done.)

- Adjust your screen contrast to make the screen very dark, then very light, and finally to suit your taste.

Grapher Notes: In this workshop, boxed items in small caps, such as $\boxed{\text{TAN}}$, suggest grapher keys. Unboxed words in small caps, such as FUNCTION MODE, suggest on-screen menu items. Most grapher keys have multiple functions. You can access the second function of a key by first pressing the special colored $\boxed{2^{\text{nd}}}$ or $\boxed{\text{SHIFT}}$ and its alphabetic function by pressing $\boxed{\text{ALPHA}}$.

Performing Calculations

Computation is done on the *Home* screen.

- Try the calculations shown in Figure 1.1. Simply key in each expression, followed by $\boxed{\text{ENTER}}$ (or $\boxed{\text{EXE}}$). To find the value of $\log 100$, use $\boxed{\text{LOG}}$. Do not enter the individual letters L, O, and G.

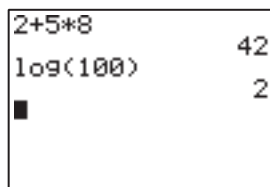


Figure 1.1

Error Messages

Don't be afraid to make a mistake! Just as pencils have erasers, graphers have delete ($\boxed{\text{DEL}}$) keys. Let's purposely make a mistake to see what happens.

- Key in $7 \boxed{\div} 0$, press $\boxed{\text{ENTER}}$, and observe your grapher's error message.

Error messages vary from grapher to grapher. Take a moment to read your grapher's error message. If a menu appears with a GoTo option, choose this option to "go to" the source of the error.

- Use your cursor keys ($\boxed{\leftarrow}$, $\boxed{\rightarrow}$, $\boxed{\uparrow}$, and $\boxed{\downarrow}$) and $\boxed{\text{DEL}}$ to change the expression from $7 \div 0$ to $7 \div 2$. Then enter this new expression.

Did you obtain the expected answer? If not, check your grapher owner's manual (CYGOM). (See the note below.) Take a few minutes to play with the editing features of your grapher. These few minutes could save you hours in the long run.

Grapher Note: Keying sequences and other procedures vary somewhat from grapher to grapher, causing a need for you to "check your grapher owner's manual." We will abbreviate this instruction as CYGOM.

Example 1 Replaying and Editing a Computation

If you deposit \$500 in a savings account at a 4.5% interest rate, compounded annually, how much will you have in your account at the end of 2, 4, and 11 years?

Solution The total value S of the investment at the end of n years is

$$S = P(1 + r)^n,$$

where r is the interest rate. Because $4.5\% = 0.045$, $1 + r = 1.045$.

- Enter the expression 500×1.045^2 on the Home screen of your grapher. If your answer has unwanted decimal places, change your *display mode* to two decimal places (CYGOM if necessary) and reenter the expression, as shown in Figure 1.2.

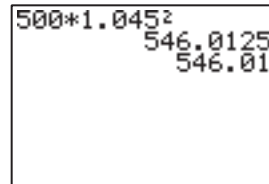


Figure 1.2 The compound interest expression computed in *floating point mode* and two decimal place *display mode*.

- *Replay* the previous entry. That is, reenter the expression without retyping the entire expression (CYGOM if necessary). Then change the exponent from 2 to 4. Repeat for the exponent 11, as shown in Figure 1.3.

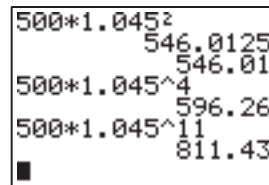


Figure 1.3 The compound interest expression computed for three different time values.

We conclude that the values in the account are \$546.01 at the end of 2 years, \$596.26 after 4 years, and \$811.43 after 11 years.

Scientific Notation

The U.S. national debt as of 1993 was \$4,351,200,000,000.

- Enter this number of dollars on the Home screen.

Your grapher will return the value in *scientific notation* because the number is so large. (See Figure 1.4.) We interpret this result as 4.3512×10^{12} dollars or, because 10^{12} is 1 trillion, as \$4.3512 trillion.

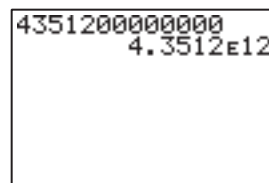


Figure 1.4

The ANS Feature

When doing a series of calculations, you can easily use the answer from one calculation in the next calculation.

- Carry out the calculations shown in Figure 1.5 by pressing $7 \times 7 \text{ [ENTER]}$, then $\text{[ANS]} \times 9 \text{ [ENTER]}$, and finally $\sqrt{\text{[ANS]}} \text{ [ENTER]}$. Note in the second step of the calculation that “Ans” automatically appeared on the screen because the grapher needed a quantity to multiply by 9.

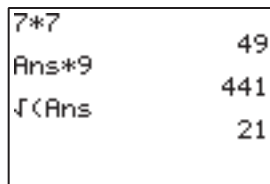


Figure 1.5

When repeating a calculation recursively, you can use the ANS feature in an extremely useful way.

- Calculate a few terms of the geometric sequence that begins with 3 and grows by a constant factor of 5 by carrying out the calculation shown in Figure 1.6.

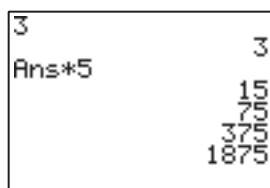


Figure 1.6

Using Variables

Another extremely helpful computational feature of the grapher is the ability to store and recall numbers as variables. The next activity illustrates using the variable x to evaluate a function at several different values of x . The activity also uses $[\text{:}]$ to string together commands. If your grapher does not have $[\text{:}]$, use [ENTER] instead.

Evaluate $f(x) = x^2 + x - 2$ at $x = 1$ by pressing

- $1 \text{ [STO]} \rightarrow \text{[X,T,θ,n]} [\text{:}] \text{[X,T,θ,n]} \text{[X}^2 \text{] [+] [X,T,θ,n] [-] 2 [ENTER]}$ for the TI-83 and TI-84, or
- $1 \text{ [STO]} \rightarrow \text{[X]} [\text{:}] \text{[X]} \text{[^] 2 [+] [X]} \text{[-] 2 [ENTER]}$ for the TI-89.

Then use the replaying and editing features of your grapher to evaluate $f(6)$ and $f(-8)$, as shown in Figure 1.7.

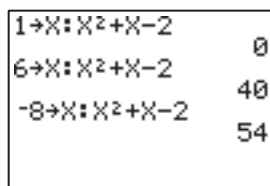


Figure 1.7

1.3 Table Building

A grapher feature even more powerful than the ability to store variables is the ability to store functions. This feature is the basis of graphing and table building. In either case the $Y = \text{edit screen}$ is used to store the symbolic expressions (rules) for functions. A table can be used to evaluate a function for several different x -values.

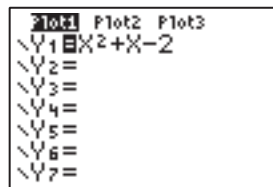
For the TI-83 and TI-84,

- press $\boxed{Y=}$ (or $\boxed{[SYMB]}$) to go to the $Y =$ screen. Then press $\boxed{X,T,\theta,n} \boxed{x^2} \boxed{+} \boxed{X,T,\theta,n} \boxed{-} \boxed{2} \boxed{[ENTER]}$. See Figure 1.8a.
- press $\boxed{[TBLSET]} \boxed{0} \boxed{[ENTER]} \boxed{1} \boxed{[ENTER]} \boxed{[ENTER]}$. See Figure 1.8b. Then press $\boxed{[TABLE]}$. See Figure 1.8c.

For the TI-89 Titanium, enter the $Y =$ Editor from the home screen, then press

$\boxed{X} \boxed{^{\wedge}} \boxed{2} \boxed{+} \boxed{X} \boxed{-} \boxed{2} \boxed{[ENTER]}$.

- Press $\boxed{[TBLSET]} \boxed{0} \boxed{\nabla} \boxed{1} \boxed{[ENTER]}$. Then press $\boxed{[TABLE]}$.



(a)



(b)

X	Y1	
0	-2	
1	0	
2	4	
3	10	
4	18	
5	28	
6	40	

Y1=X²+X-2

(c)

Figure 1.8 The steps in the table-building process on a grapher.

- Use the cursor keys ($\boxed{\blacktriangleright}$, $\boxed{\blacktriangleleft}$, $\boxed{\blacktriangleup}$, and $\boxed{\blacktriangledown}$) to move around the table and explore. Pay attention to the readout at the bottom of the screen as you move to different “cells” in the table. What happens when you try to move the cursor off the top or bottom of the screen?

1.4 Function Graphing

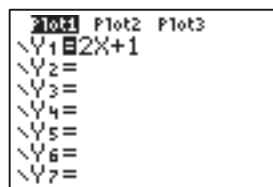
Graphing and Exploring a Function

Most graphers have several graphing modes. Be sure that your grapher is in FUNCTION mode. The algebraic form for the function needs to be $f(x) = \dots$, or $y = \dots$.

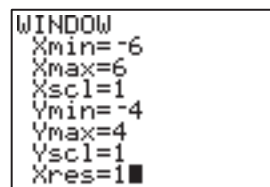
Example 2 Graphing and Tracing Along a Function

Use the FUNCTION mode to graph $f(x) = 2x + 1$. Explore the ordered pairs of the graph with the TRACE feature.

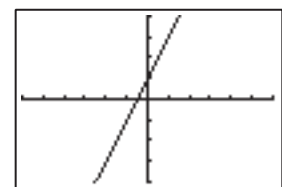
Solution Figure 1.9 illustrates the procedure for graphing f in the window $[-6, 6]$ by $[-4, 4]$. Window is called range on some graphers.



(a)



(b)



(c)

Figure 1.9 The steps in the graphing process.

- Enter $y = 2x + 1$ as shown in Figure 1.9a and the *window dimensions* as shown in Figure 1.9b. Then press [GRAPH] or [PLOT] to obtain the graph shown in Figure 1.9c.
- Press [TRACE] (F3 on the TI-89) to display x - and y -coordinates of points on the graph. Use [▶] or [◀] to move from point to point. See Figure 1.10. This *tracing* shows you which points of $f(x) = 2x + 1$ were plotted by the grapher.

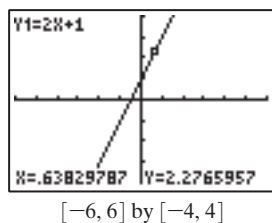


Figure 1.10 The graph of $f(x) = 2x + 1$ with the TRACE feature activated.

- Change the view dimensions to $[-10, 10]$ by $[-10, 10]$, known as the *standard window*. Some graphers have a [ZOOM] key (F2 on the TI-89). If yours does, press [ZOOM], then choose ZSTANDARD from the menu to set the window to $[-10, 10]$ by $[-10, 10]$ automatically. Press [TRACE] and explore. Are the (x, y) pairs the same as you found using the window $[-6, 6]$ by $[-4, 4]$?

A grapher allows you to obtain several views of the graph of a function. The Xmin and Xmax window dimensions determine which points the grapher plots and hence the coordinate readout when the TRACE feature is activated.

Your choice of Xmin and Xmax affects the x -coordinate readout when you trace along a graph. The reason is that the grapher screen is a rectangular array of *pixels*, short for “picture elements.” The change in x -value that occurs when tracing is given by

$$\Delta x = \frac{X_{\max} - X_{\min}}{\text{Number of columns of pixels} - 1}$$

The number of columns of pixels varies from grapher to grapher, as indicated in Table 1.1.

Table 1.1 The number of columns of pixels on various graphers

Grapher	Columns of Pixels
TI-80	63
Casio, Sharp, TI-82, TI-83	95
TI-81	96
TI-85	127
Hewlett-Packard	131
TI-92	239

FRIENDLY WINDOWS

As we observed in Example 2, the x -values displayed on the screen during tracing have many decimal places—for example, $x = 0.63829787$, as shown in Figure 1.10. Such “unfriendly” x -values can be avoided. You can use the [Xmin, Xmax] settings given in Table 1.2, or positive integer multiples of these settings, to guarantee a *friendly* x -coordinate readout when tracing. Windows with friendly x -coordinates are called *friendly windows*.

Table 1.2 The [Xmin, Xmax] dimensions for a basic friendly window on various graphers

Grapher	[Xmin, Xmax]
TI-80	[-3.1, 3.1]
Casio, Sharp, TI-82, TI-83	[-4.7, 4.7]
TI-81	[-4.8, 4.7]
TI-85	[-6.3, 6.3]
Hewlett-Packard	[-6.5, 6.5]
TI-92	[-11.9, 11.9]

Grapher Note: Some graphers have a ZDECIMAL feature that sets [Xmin, Xmax] to the basic friendly settings and a ZINTEGER feature that can set the x -view dimensions to 10 times the basic friendly settings.

- Graph the function $f(x) = 2x + 1$ from Example 2 in a friendly window, using the [Xmin, Xmax] settings given for your grapher in Table 1.2. Trace along the graph.
- Double the [Xmin, Xmax] and [Ymin, Ymax] settings and trace along the new view of the graph. How has the x -coordinate readout changed?
- Enter 10 times the settings given for your grapher in Table 1.2 to obtain *integer settings*. Trace to learn the reason for this name.

Square Windows

The graph of $f(x) = 2x + 1$ is a straight line with a slope of 2. You have seen several views of this graph in different windows. The apparent steepness of the graph can be quite different even though the slope is always 2.

- Graph $f(x) = 2x + 1$ in the window $[-9, 9]$ by $[-2, 2]$ and then in the window $[-9, 9]$ by $[-20, 20]$. Compare the apparent steepness of the graph in the two windows.

In general, to obtain a graph that suggests the graph's true shape, you must choose viewing dimensions that are proportional to the dimensions of your grapher screen. Most grapher screens have a width-to-height ratio of roughly 3 : 2. Windows whose dimensions are proportional to the physical dimensions of the grapher screen are called *square windows*. Square windows yield true shapes: They make perpendicular lines look perpendicular, squares look square, and circles look circular.

Example 3 Rounding Out a Circle

Use your grapher to plot the circle $x^2 + y^2 = 1$.

Solution First, you will need to do some algebra:

$$\begin{aligned}x^2 + y^2 &= 1 \\y^2 &= 1 - x^2 \\y &= \pm\sqrt{1 - x^2}\end{aligned}$$

So the graph of the circle $x^2 + y^2 = 1$ is the union of the graphs of the functions

$$y_1 = \sqrt{1 - x^2} \text{ and } y_2 = -\sqrt{1 - x^2}.$$

- Graph $y_1 = \sqrt{1 - x^2}$ and $y_2 = -\sqrt{1 - x^2}$ in several windows with different x - y dimension ratios. (We used parentheses around $1 - x^2$ because you will need them to enter the functions onto the Y = edit screen.) Continue until you obtain a graph that appears circular. *Note:* Gaps may appear near the x -axis.
- Graph $y_1 = \sqrt{1 - x^2}$ and $y_2 = -\sqrt{1 - x^2}$ in a square, friendly window.

Grapher Note: Some graphers have a ZSQUARE feature that adjusts the window dimensions to make them match the physical proportions of the screen.

Figure 1.11 shows three views. Only the view in part (a) looks circular because only in part (a) are the window dimensions proportional to the physical dimensions of the screen.

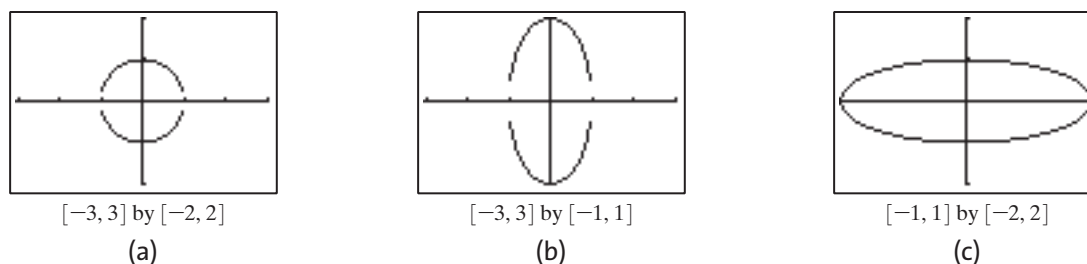


Figure 1.11 The graph of a circle in (a) a square window and (b and c) nonsquare windows.

Square, friendly, or standard windows are not always ideal. Example 4 illustrates that window dimensions may need to be exaggerated to reveal the important features of a graph.

Example 4 Finding Key Features of a Graph

Graph $f(x) = x^2(x + 7)^3$, using $X_{\min} = -10$ and $X_{\max} = 10$. Try various $[Y_{\min}, Y_{\max}]$ settings.

Solution Figure 1.12 shows three possible views.

- Try the windows shown in Figure 1.12 and several others of your own choosing. Continue until you convince yourself that you have observed all the important features of $f(x) = x^2(x + 7)^3$ on the interval $[-10, 10]$.

Note that the view in part (c) shows more features of the graph than either part (a) or part (b) and hence is the best view of the three. Further investigation of the graph in other windows should reveal no other major features on the interval $[-10, 10]$.

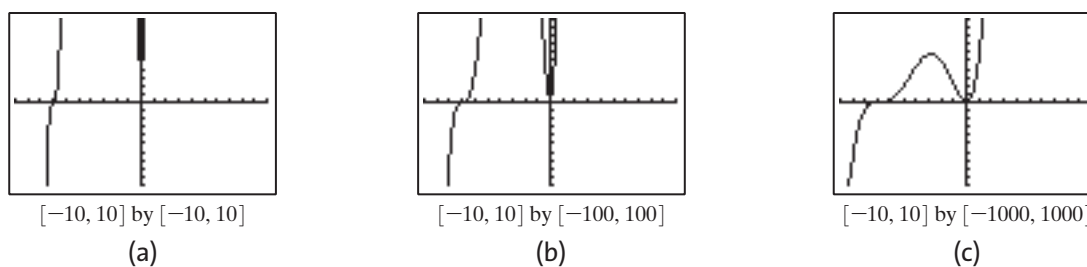


Figure 1.12 The graph of the same function in three different viewing windows.

Example 5 is designed to familiarize you with 12 basic graphs that are used in the textbook and are important as models in many fields of endeavor. You should learn these graphs by heart! That is, you should be able to sketch any of them quickly at any time without a great deal of thought, without doing any hand computation, and without using your grapher.

Example 5 Touring a Gallery of Basic Functions

Plot and explore the 12 graphs shown in Figure 1.13.

Solution You may need to do some searching on your grapher keyboard or dig through some menus to find all these functions (CYGOM). You should use function notation—for example, $\text{abs}(X)$ rather than $\text{abs } X$ —even though your grapher may be forgiving.

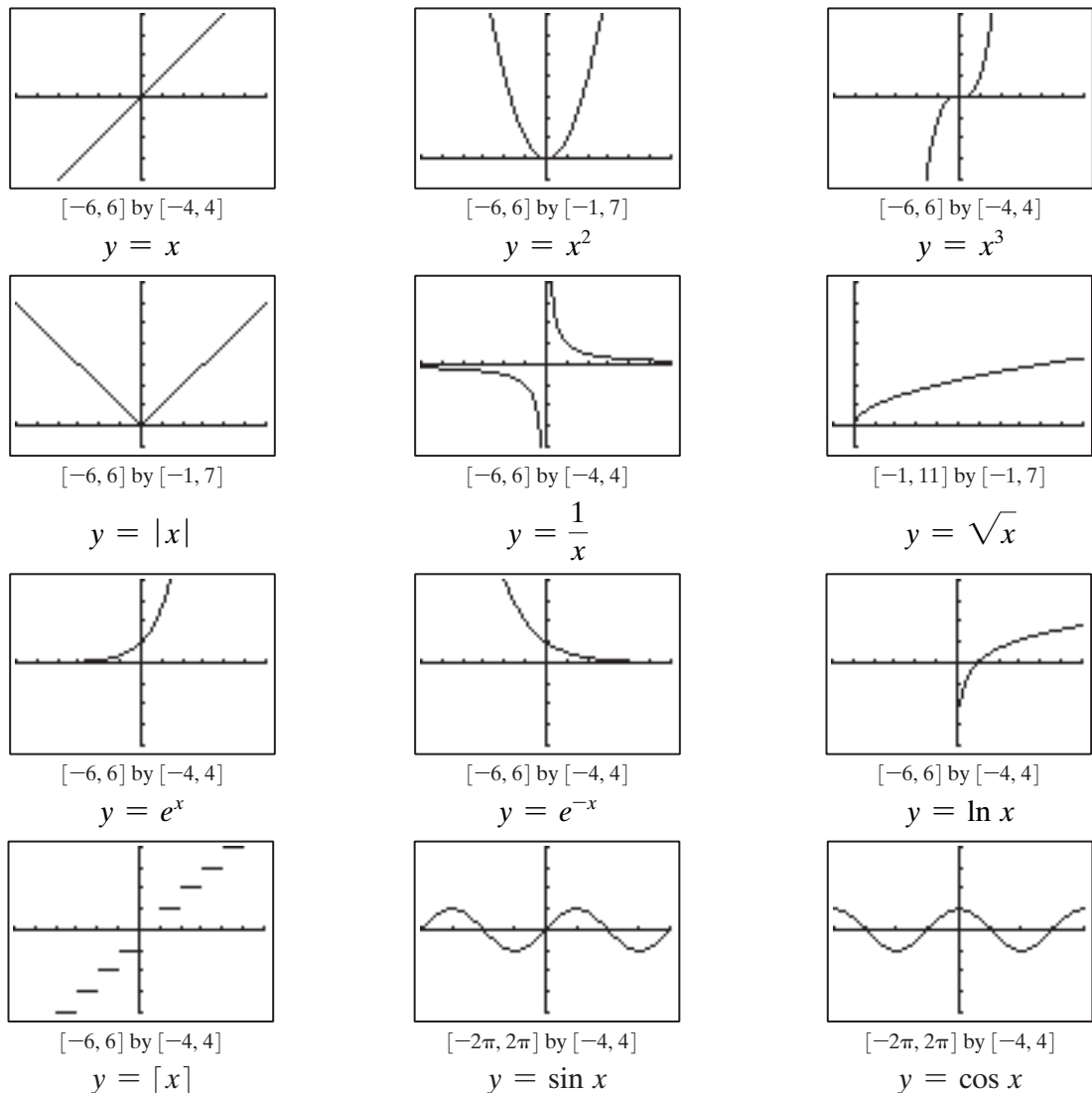


Figure 1.13 Gallery of Basic Functions

- Graph all but the sine and cosine functions with the windows indicated in Figure 1.13. Regraph each function for other window dimensions, including standard, friendly, and square. Explore with TRACE.
- Graph the greatest integer function in CONNECTED mode and then in DOT mode. Which produces the better graph? Why?
- Graph the sine and cosine functions in DEGREE mode and then in RADIAN mode. Note that the graphs in Figure 1.13 are based on the use of RADIAN mode.

Grapher Note: Some graphers have ZTRIG that automatically sets desirable window dimensions for viewing trigonometric functions.

You should practice making hand-drawn sketches of the 12 basic functions until you can do them from memory.

1.5 Graphical Problem Solving

In this section we explore various grapher methods for solving equations and analyzing the graphical behavior of functions, so you should set your grapher to FUNCTION mode. We begin by showing how to solve equations graphically, using the example

$$|x| = x^2 + x - 2,$$

first by graphing

$$f(x) = |x| \quad \text{and} \quad g(x) = x^2 + x - 2$$

separately and then by investigating the related function

$$h(x) = f(x) - g(x) = |x| - (x^2 + x - 2).$$

Solving an Equation by Finding Intersections

We can solve an equation by graphing each side as a function and locating the points of intersection.

- Enter each side of the equation $|x| = x^2 + x - 2$ onto the Y= edit screen as shown in Figure 1.14.
- Graph the equations in a friendly window. See Figure 1.15.

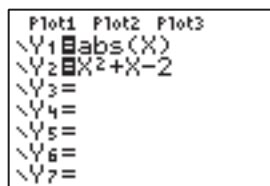
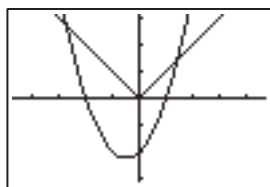


Figure 1.14



$[-4.7, 4.7]$ by $[-3.1, 3.1]$

Figure 1.15 The dimensions of a friendly window vary from grapher to grapher. Possible x dimensions are given in Table 1.2. ZDECIMAL or ZINTEGER yield a square, friendly window on some graphers.

The graph of f is V-shaped and the graph of g is an upward opening parabola. They have two points of intersection. Thus the equation $|x| = x^2 + x - 2$ has two solutions. These solutions are the x -coordinates (one positive and one negative) of the two points of intersection.

- Trace along either graph to approximate the positive solution, that is, to estimate the x -coordinate of the point of intersection in the first quadrant.

To get a better approximation for the positive solution we can *zoom in* by picking smaller and smaller windows that contain the point of intersection in the first quadrant. Three common ways to zoom in are to

1. change the WINDOW settings manually;
2. use ZOOMBOX, which lets you use the cursor to select the opposite corners of a “box” to define a new window; and

3. use ZOOMIN, which magnifies the graph around the cursor location by a factor that you can set.
- Practice each of the three types of zooming (CYGOM if necessary). Trace after each zoom step, as shown in Figure 1.16.

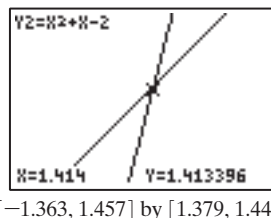


Figure 1.16 One possible view of the graphs after some zooming, with the TRACE feature activated.

Most graphers have an INTERSECTION feature that can be used to automate the process of solving equations graphically without adjusting the viewing window dimensions.

- Graph the equations of Figure 1.14 in a friendly window (Figure 1.15). Then use the INTERSECTION feature to locate the point of intersection in the first quadrant, as shown in Figure 1.17.

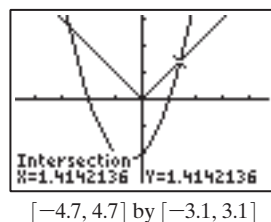


Figure 1.17

Solving the equation $|x| = x^2 + x - 2$ algebraically reveals that the positive solution is $x = \sqrt{2} = 1.4142\dots$, which confirms the graphical solution.

Solving by Finding x-Intercepts

To solve an equation of the form $f(x) = g(x)$, we can solve $f(x) - g(x) = 0$. Then the problem becomes one of finding where the functions $y = f - g$ and $y = 0$ intersect, or simply the x -intercepts of $y = f - g$. For example, to solve the equation $|x| = x^2 + x - 2$, we can find the x -intercepts of

$$y = h(x) = f(x) - g(x) = |x| - (x^2 + x - 2).$$

- Load the Y= edit screen, as shown in Figure 1.18a, selecting only $y_3 = y_1 - y_2 = \text{abs}(x) - (x^2 + x - 2)$ to be graphed.
- Graph y_3 in a friendly window, as shown in Figure 1.18b.
- The x -intercepts are also the “zeros” of the equation. Use the ZERO feature (CYGOM if necessary) to locate the negative x -intercept, as shown in Figure 1.18c.

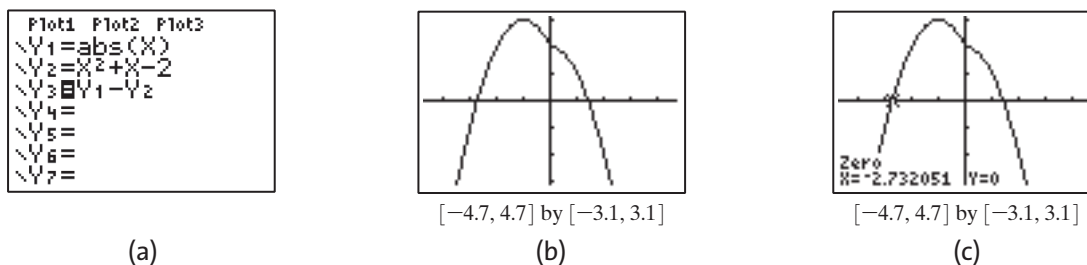


Figure 1.18 Finding a solution of an equation by finding an x -intercept of the difference function.

Studying Graph Behavior

As you have seen, TRACE allows you to move from pixel to pixel on a graph with the coordinates of the points displayed to illustrate the numerical behavior of the function. For example, you can see whether the y -coordinate increases, decreases, or remains constant as x increases. ZOOMIN permits a “close-up” examination of the *local behavior* of graphs.

Three other grapher features are useful for investigating graph behavior (CYGOM if necessary).

1. VALUE evaluates a function for a given domain value, which often avoids the need for a friendly window.
2. MINIMUM finds a local minimum value of a function and the associated domain value.
3. MAXIMUM finds a local maximum value of a function and the associated domain value.

Example 6 Investigating Graph Behavior

Graph $f(x) = x^3 + 2x^2 - 5x - 6$ and study its behavior.

Solution Do the following on your grapher.

- Graph $y = f(x)$ in the standard window. Trace over until $x \approx -3.4$, as shown in Figure 1.19a. Then trace from left to right. Observe whether the function values (y -coordinates) increase, decrease, or remain constant as x increases.
- The graph appears to show that $f(2) = 0$, but TRACE fails to give a y value for $x = 2$. Use VALUE to find $f(2)$, as shown in Figure 1.19b.
- To determine precisely the intervals on which f is increasing or decreasing, locate the domain values at which the local maximums and minimums occur. Use MAXIMUM and MINIMUM to find these values. Figure 1.19c shows the result of using the MAXIMUM feature.

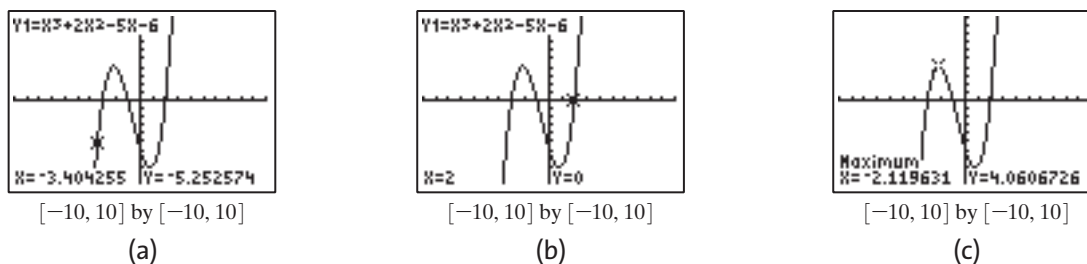


Figure 1.19 Exploring a graph with (a) TRACE, (b) VALUE, and (c) MAXIMUM.

- Zoom in around the points that correspond to the local maximum and the local minimum. Describe the graph's behavior near each of these points.

What other behaviors of graphs can you study with the features of your grapher?

1.6 Parametric and Polar Graphing

Parametric Graphing

To graph parametric equations, set your grapher to PARAMETRIC mode. In PARAMETRIC mode, pressing $\boxed{X,T,\theta,n}$ will yield the independent variable t . Note that the TI-89 Titanium has a separate key for each variable. (CYGOM if necessary.)

The parametric equations

$$x(t) = \dots, \quad y(t) = \dots$$

are entered in the form $x_1(t) = \dots, y_1(t) = \dots$ on the Y= edit screen.

Example 7 Graphing Parametric Equations

Graph the parametric equations.

$$x = t^2, \quad y = t - 1 \quad \text{for} \quad -2 \leq t \leq 2$$

Solution Follow these steps:

- Enter the parametric equations on the Y= edit screen, as shown in Figure 1.20.

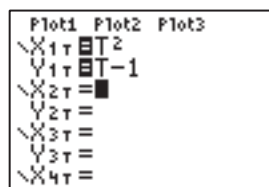


Figure 1.20

- Set the WINDOW dimensions shown in Figure 1.21. You will need to scroll down to see the entire menu given in Figure 1.21 because it has too many lines.

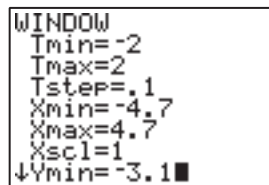


Figure 1.21 Facsimile of the WINDOW screen set for plotting the parametric equations.

Tstep on the parametric WINDOW menu sets the step size between the successive t -values that the grapher uses to compute and plot (x, y) pairs. In this case, the Tstep of 0.1 will yield 40 steps from the Tmin of -2 to the Tmax of 2 . Thus 41 points will be calculated and plotted, with the points corresponding to

$$t = -2.0, -1.9, -1.8, -1.7, \dots, 1.9, 2.0.$$

- Press $\boxed{\text{GRAPH}}$ or $\boxed{\text{PLOT}}$ to obtain the graph shown in Figure 1.22.



Figure 1.22 The graph in (a) CONNECTED mode and (b) DOT mode.

- Use TRACE to explore the graph numerically. Note that the values of the parameter t and the coordinates x and y are all shown on the screens in Figure 1.23.



Figure 1.23 Two views of the parametric curve with TRACE activated.

Figures 1.22 and 1.23 show only a piece of the parabola $x = (y + 1)^2$ rather than the complete parabola for this viewing window. The parametric WINDOW menu allows you to choose a part of the graph by setting Tmin and Tmax. Do you see why? If not, experiment with the Tmin and Tmax settings until you do.

Polar Graphing

To graph polar equations set your grapher to POLAR mode. Pressing $\boxed{x, T, \theta, n}$ will yield the independent variable θ .

Example 8 Graphing Polar Equations

Simultaneously graph $r = 9 \sin 5\theta$ and $r = 9$.

Solution Follow these steps:

- Set your grapher to SIMULTANEOUS and RADIAN modes. Then enter the polar equations on the Y= edit screen, as shown in Figure 1.24.

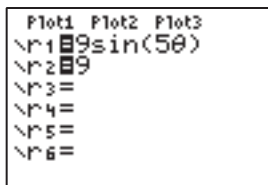


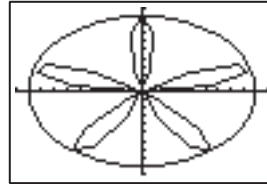
Figure 1.24

- Set the WINDOW dimensions to

$$\theta_{\min} = 0, \quad \theta_{\max} = 2\pi, \quad \theta_{\text{step}} = \pi/24,$$

using the standard dimension of $[-10, 10]$ by $[-10, 10]$ for x and y . (On some graphers, you can obtain these settings by using ZSTANDARD.)

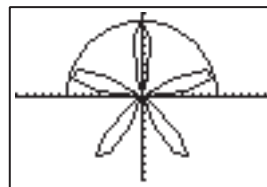
- Press [GRAPH] or [PLOT] to obtain the graphs shown in Figure 1.25.



$[-10, 10]$ by $[-10, 10]$

Figure 1.25 The circle $r = 9$ and the 5-petaled rose $r = 9 \sin 5\theta$.

- Trace along the two polar curves.
- Set $\theta \text{ max} = \pi$. Then use ZSQUARE to square the viewing window. If no such command is available on your grapher, reset the x -dimensions of the WINDOW menu by hand. Figure 1.26 shows the result.



$[-15, 15]$ by $[-10, 10]$

Figure 1.26

Note that the entire rose curve is plotted for the interval $0 \leq \theta \leq \pi$. Explore the effect of changing the 5 in the equation $r = 9 \sin 5\theta$ to another number.

1.7 Curve Fitting and Statistics

A grapher can help you organize, process, and analyze data, as well as compute and plot models for paired data. The procedures for data analysis and curve fitting vary a great deal from grapher to grapher (CYGOM for details).

Example 9 Plotting and Fitting Data

Plot the national debt data given in Table 1.3, find a model for the data, and then overlay a graph of the model on the scatter plot.

Table 1.3 U.S. Public Debt, 1950–1990

Year	Debt (Billions of dollars)
1950	256.1
1960	284.1
1970	370.1
1975	533.2
1980	907.7
1985	1,823.1
1990	3,233.3

Source: *The World Almanac and Book of Facts* (1995, Mahwah, N.J.: Funk & Wagnalls), p. 109.

Solution Follow these steps:

- Enter the data shown in Table 1.3 into the statistical memory of your grapher, as shown in Figure 1.27a.
- Set an appropriate window for the data, letting x be the year and y be the amount of the debt, as shown in Figure 1.27b.
- Make a scatter plot of the data, as shown in Figure 1.27c.

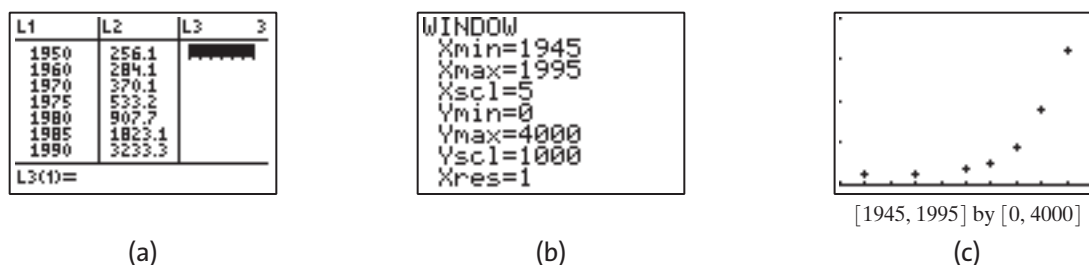


Figure 1.27 The steps involved in making a scatter plot on a grapher.

Most graphers have several regression options. Typically, linear, quadratic, exponential, logarithmic, and power functions are available as regression models. Some graphers offer other options. The following activity illustrates quadratic regression. That is, we find a quadratic function that closely fits the given data.

- Choose the QUADRATIC option from your grapher's regression menu. Use the first column of data for the x -values and the second column as y -values. The grapher should return approximate coefficients for the quadratic regression equation, as shown in Figure 1.28a.
- Load the regression equation onto the $Y=$ edit screen, as shown in Figure 1.28b.
- Press [GRAPH] or [PLOT] to obtain the graph in Figure 1.28c, which shows both the scatter plot and the graph of the regression equation.

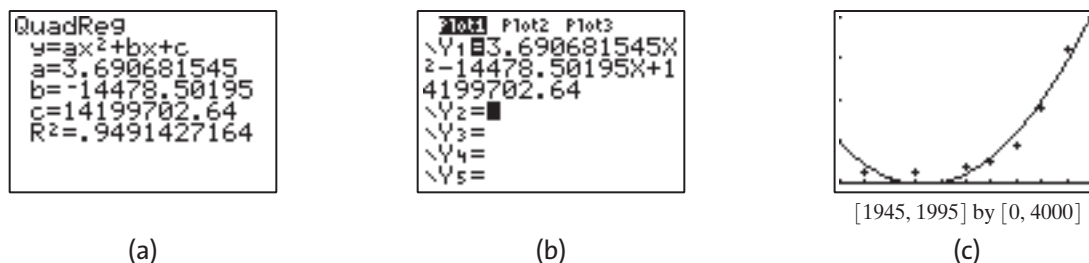


Figure 1.28 The steps involved in fitting a quadratic function to a scatter plot.

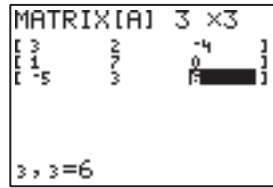
Repeat the steps to find and graph other regression equations for the data.

Most graphers offer a variety of statistical plots, often including histograms, boxplots, and line graphs. In addition, graphers can carry out many types of statistical computations (CYGOM for details).

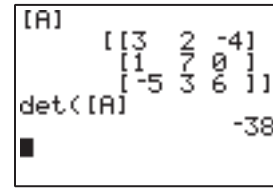
1.8 Matrix Calculations

A grapher can perform many matrix operations, thus avoiding the tedium of hand computation. Matrix procedures vary somewhat from grapher to grapher (CYGOM for details).

- Place matrix A on the Home screen (Figure 1.29a) and compute its determinant (Figure 1.29b).




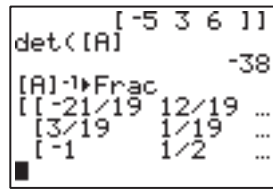
(a)



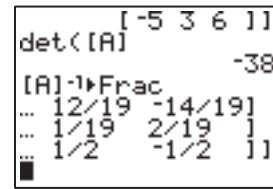
(b)

Figure 1.29 (a) Matrix entry and (b) matrix display and determinant calculation.

- Compute the inverse of matrix A . You may need to scroll, using  in order to see the entire answer (Figure 1.30).



(a)

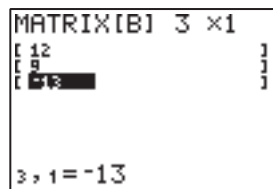


(b)

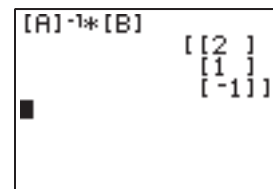
Figure 1.30 (a) Computing the inverse of matrix A and (b) scrolling.

Grapher Note: In Figure 1.30a, we used the FRAC feature to convert the decimal entries of A^{-1} to fractions.

- Enter the matrix $B = \begin{pmatrix} 12 \\ 9 \\ -13 \end{pmatrix}$ into the matrix editor of your grapher (Figure 1.31a).
- Compute the product $A^{-1} \cdot B$ on the Home screen (Figure 1.31b).



(a)



(b)

Figure 1.31 (a) Entering a column matrix and (b) multiplying matrices.

Graphers have other matrix features. Most importantly, many graphers can perform elementary row operations on a matrix.

1.9 Grapher Insights and Caveats

Limitations of Grapher Computations

Grapher computations are limited in *magnitude* and *relative accuracy*. Numbers less than the lower magnitude limit are rounded to zero. Numbers greater than or equal to the upper magnitude limit yield *overflow errors*.

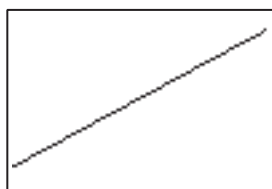
Regarding relative error, some graphers store only the first 13 significant digits of a number (the rest of the number is rounded off) and they display at most 10 digits. To see this in action,

- Enter $(2/3 - 1/3) - 1/3$ on your grapher.

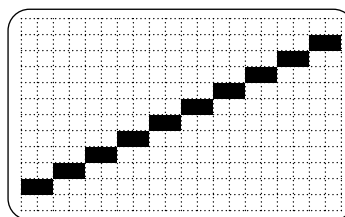
We know that $(2/3 - 1/3) - 1/3 = 0$, but older graphers will not give 0 as the final result. What steps in the grapher computation could have produced this result? Explain.

How a Grapher Draws a Function Graph

All graphers produce graphs, or plots, by lighting pixels on a liquid crystal display (LCD). Figure 1.33a shows the graph of a line on an LCD, and Figure 1.33b shows how it might appear under a magnifying glass. When the grapher is in `CONNECTED` mode, the plotted pixels are connected by “line segments” of pixels. When in `DOT` mode, the pixels are left unconnected.



(a)



(b)

Figure 1.33 (a) The graph of a line on an LCD and (b) how it looks under magnification.

Limitations of Grapher Plotting

Grapher plotting is limited by roundoff error in the calculation of the y -coordinates from the function formula. More significantly, grapher plotting is limited because, for each point plotted, a pixel is rarely centered on the LCD at the exact vertical location corresponding to the y -coordinate of the point.

Despite these limitations, graphers can quickly produce accurate graphs for most functions described in this book if the viewing dimensions are chosen appropriately. It takes mathematical and grapher experience to get good at choosing windows. You will develop this skill over time as you go through the workshop exercises and the rest of the book.

Interpreting Grapher Plots

As Examples 3 and 4 illustrated, a grapher plot may be misleading or incomplete. (See Figures 1.11 and 1.12.) True visualization occurs in the “mind’s eye” when you use the information gained from a grapher together with your mathematical knowledge to obtain a mental image of the mathematical graph. To communicate this mental visualization to another person, however, you must learn to describe and draw graphs accurately. When recording a graph on paper, you should normally add suggestive arrowheads and label axes, key points, and other pertinent information. Figure 1.34 shows how two rather misleading views of a complicated graph can be combined into a single recorded graph. This is a skill to strive for as you work with the grapher.

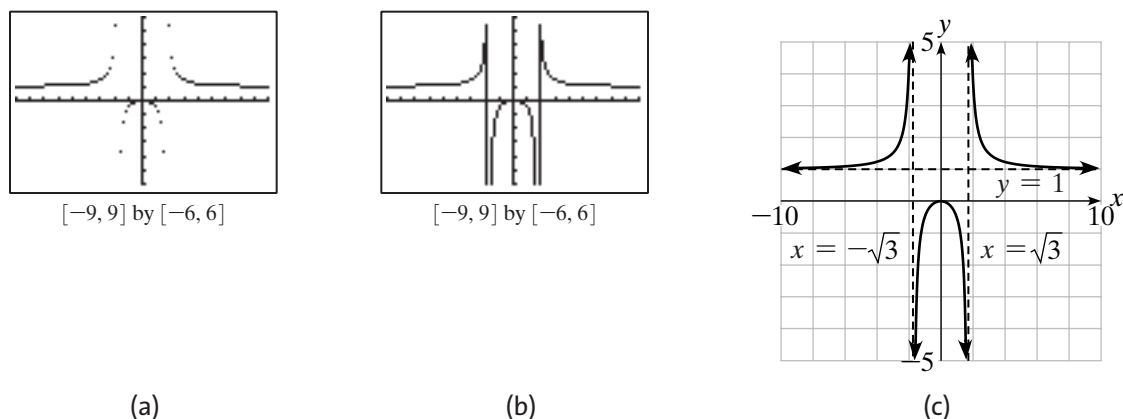


Figure 1.34 The graph of $y = x^2/(x^2 - 3)$ as shown (a and b) on a grapher and (c) on paper.

1.10 Viewing Window Summary

Choosing a Viewing Window

You need experience and mathematical expectations to choose appropriate viewing windows. One approach is to start with the *standard window* of $[-10, 10]$ by $[-10, 10]$ and adjust the y -dimensions.

Some windows may show more features of a graph than others. The view on the right shows the key features of the graph; the other two views do not.

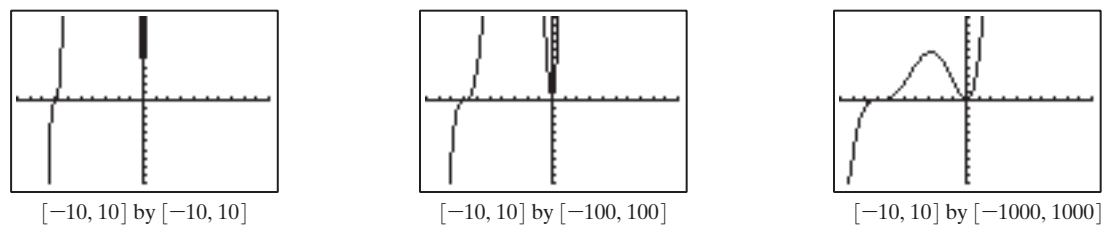


Figure 1.35 Three views of $f(x) = x^2(x + 7)^3$.

Friendly Windows

Your choice of X_{\min} and X_{\max} affect the x -coordinate readout when you *trace* along a graph. You can use the $[X_{\min}, X_{\max}]$ settings given in the table, or positive integer multiples of these settings, to guarantee a *friendly* x -coordinate readout when tracing. Windows with friendly x -coordinates are called *friendly windows*.

Table 1.4 The $[X_{\min}, X_{\max}]$ dimensions for a basic friendly window on various graphers.

Grapher	$[X_{\min}, X_{\max}]$
TI-80	$[-3.1, 3.1]$
Casio, Sharp, TI-82, TI-83	$[-4.7, 4.7]$
TI-81	$[-4.8, 4.7]$
TI-85	$[-6.3, 6.3]$
Hewlett-Packard	$[-6.5, 6.5]$
TI-92	$[-11.9, 11.9]$



Figure 1.36 Graphs of $f(x) = \sqrt[3]{x}$

Square Windows

A *square window* is a window that shows the true shape of a graph. Such a window makes perpendicular lines look perpendicular, a square look square, and a circle look circular. A square window has the same proportions as your grapher screen. Many grapher screens have a width-to-height ratio of 3 : 2. Most graphers have a built-in feature for squaring windows.

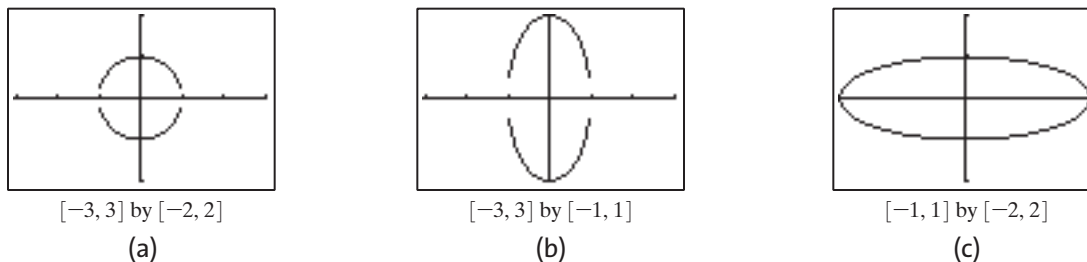


Figure 1.37 Three views of the circle $x^2 + y^2 = 1$.

Integer and Decimal Windows

These windows are special types of friendly windows. An *integer window* is a window in which each pixel is centered at a point with integer coordinates and the change in both x and y is 1. A *decimal window* is a window in which each pixel is centered at a point with coordinates of at most one decimal place and the change in both x and y is 0.1. On most graphers, both integer and decimal windows are square and friendly.

1.11 Exercises for Chapter 1

The exercises are correlated with the sections of the *Grapher Workshop*. If any give you difficulty, resolve it by consulting either your owner's manual or your instructor.

Numerical Computation and Editing

In Exercises 1–6, use a grapher to evaluate the expression.

1. 600×1.075^2
2. 1.3^5
3. $\log(1/10)$
4. $|-7|$
5. $\sqrt[3]{64}$
6. $\sqrt[4]{81}$

In Exercises 7 and 8, use ANS.

7. Calculate the first seven terms in the geometric sequence that begins with 2 and grows by a constant factor of 6: 2, 12, 72,
8. Calculate the first seven terms in the arithmetic sequence that begins with 4 and grows by a constant 9: 4, 13, 22,

In Exercises 9 and 10, use replay.

9. Evaluate $f(x) = x^2 + x - 2$ at $x = -3, -2.5, 3$.
10. Evaluate $g(t) = 2t^3 - |5t|$ at $t = -2, 3.4, 7$.

Table Building

In Exercises 11–14, make a table with the following inputs.

- a. $x = -3, -2, -1, \dots, 3$
- b. $x = 0, 1, 2, \dots, 6$
- c. $x = -10, -5, 0, \dots, 20$
11. $f(x) = x^2 + 15$
12. $f(x) = \ln(x^2 + 1)$
13. $f(x) = |x - 2|$
14. $f(x) = \sqrt[3]{x^2 + x - 2}$

Function Graphing

In Exercises 15–20, graph the function in each type of window.

- a. The standard window
- b. A square window containing the standard window
- c. A friendly window using TRACE to support the friendly x -coordinate readout
- d. A window that is both square and friendly
15. $y = 3x - 2$
16. $y = -\frac{1}{2}x + 3$
17. $y = 1 - x^2$
18. $y = 2x^2 - 3x + 1$
19. $y = |x + 2|$
20. $y = \sqrt{9 - x^2}$

In Exercises 21 and 22, explain any differences in the two modes.

21. Enter and graph

$$y_1 = 2x + 1;$$

$$y_2 = 2x + 2;$$

$$y_3 = 2x + 3.$$

Switch between SEQUENTIAL and SIMULTANEOUS modes and draw the graphs again.

22. Graph $y = 3x - 1$. Switch between CONNECTED and DOT modes and draw the graph again.

In Exercises 23 and 24, find a window that reveals the key features of the function. How many x -intercepts does the graph have?

23. $y = x^2(x - 12)$
 24. $y = x(x + 3)^2(x + 14)$

Graphical Problem Solving

In Exercises 25–28, solve the equation by

- finding intersections.
- finding x -intercepts.

25. $|x| = \frac{1}{2}x + 1$
 26. $|x - 3| = -\frac{1}{2}x + 4$
 27. $x - 2 = 1 - x^2$
 28. $x - 3 = x^2 - 5$

In Exercises 29 and 30, (a) find a viewing window that reveals the key features of the graph, and (b) find the local maximum and minimum values.

29. $y = x^2(x - 12)$
 30. $y = x(x + 3)^2(x + 14)$

Parametric and Polar Graphing

In Exercises 31–34, use PARAMETRIC mode.

31. Enter

$$x(t) = t, \quad y(t) = t.$$

Predict what will happen as a result of the WINDOW settings for t . Then graph and compare with your prediction.

- $T_{\min} = -10, T_{\max} = 10, T_{\text{step}} = 0.1$
- $T_{\min} = 0, T_{\max} = 10, T_{\text{step}} = 0.1$
- $T_{\min} = -10, T_{\max} = 0, T_{\text{step}} = 0.1$
- $T_{\min} = -10, T_{\max} = 10, T_{\text{step}} = 1$
- $T_{\min} = -5, T_{\max} = 5, T_{\text{step}} = 1$
- $T_{\min} = 10, T_{\max} = -10, T_{\text{step}} = -0.1$

32. Graph

$$x_1(t) = t, \quad y_1(t) = 2t;$$

$$x_2(t) = 2t, \quad y_2(t) = t,$$

with $T_{\min} = -10, T_{\max} = 10, T_{\text{step}} = 0.1$. Predict what the graphs will look like. Then graph and compare with your prediction.

33. Graph

$$x_1(t) = 2t, \quad y_1(t) = t^2,$$

with $T_{\min} = -10, T_{\max} = 10, T_{\text{step}} = 0.1$.

34. Graph

$$x_1(t) = 2t, \quad y_1(t) = t^2;$$

$$x_2(t) = y_1(t), \quad y_2(t) = x_1(t),$$

with $T_{\min} = -10$, $T_{\max} = 10$, $T_{\text{step}} = 0.1$.

In Exercises 35–38, use POLAR and RADIAN modes.

35. Graph $r = 6$ in a square window. Switch between CONNECTED and DOT modes and graph again.

36. Graph $r = 2 \cos \theta$ in a square window. Switch between CONNECTED and DOT modes and graph again.

37. Graph $r_1 = 2$ and $r_2 = 5$. Switch between SEQUENTIAL and SIMULTANEOUS modes and graph again.

38. Graph $r_1 = 2$ and $r_2 = 2 \cos 3\theta$. Switch between SEQUENTIAL and SIMULTANEOUS modes and graph again.

Curve Fitting and Statistics

In Exercises 39 and 40, use the data in Table 1.5.

Table 1.5 Official Census Population (in millions of persons), 1900–1990

Year	Florida	Pennsylvania
1900	0.5	6.3
1910	0.8	7.7
1920	1.0	8.7
1930	1.5	9.6
1940	1.9	9.9
1950	2.7	10.5
1960	5.0	11.3
1970	6.8	11.8
1980	9.7	11.9
1990	12.9	11.9

Source: U.S. Census Bureau as reported in *The World Almanac and Book of Facts* (1995, Mahwah, N.J.: Funk & Wagnalls), p. 377.

39. a. Enter the Florida population data into the statistical memory of your grapher.
 b. Set an appropriate window for the data and make a scatter plot.
 c. Choose the EXPONENTIAL option from your grapher's regression menu to find the constants in the regression equation.
 d. Load the regression equation onto the $Y =$ edit screen and overlay the graph of the regression equation on the scatter plot.
40. a. Enter the Pennsylvania population data into the statistical memory of your grapher.
 b. Set an appropriate window for the data and make a scatter plot.
 c. Choose the LINEAR option from your grapher's regression menu to find the coefficients for the linear regression equation.

- d. Load the regression equation onto the $Y =$ edit screen and overlay the graph of the regression equation on the scatter plot.

Matrix Calculations

In Exercises 41 and 42, perform the computations.

41. a. Enter the matrix

$$A = \begin{pmatrix} 1 & -3 \\ 5 & 2 \end{pmatrix}$$

into the matrix editor of your grapher and compute its determinant.

- b. Compute the inverse of matrix A .

- c. Enter the matrix

$$B = \begin{pmatrix} -1 \\ 12 \end{pmatrix}$$

into the matrix editor of your grapher and compute the product $A^{-1} \cdot B$.

42. a. Enter the matrix

$$A = \begin{pmatrix} 4 & 2 & -1 \\ 1 & -1 & 0 \\ 0 & 3 & 5 \end{pmatrix}$$

into the matrix editor of your grapher and compute its determinant.

- b. Compute the inverse of matrix A .

- c. Enter the matrix

$$B = \begin{pmatrix} 15 \\ 2 \\ -2 \end{pmatrix}$$

into the matrix editor of your grapher and compute the product $A^{-1} \cdot B$.

Grapher Insights and Caveats

In Exercises 43 and 44, sketch the graph.

43. Graph

$$y = \frac{(x + 3)^2}{x(x + 4)}$$

in CONNECTED and DOT modes and then sketch the graph on paper.

44. Graph

$$y = \frac{x(x - 3)}{(x - 1)(x + 4)}$$

in CONNECTED and DOT modes and then sketch the graph on paper.