INTERMEDIATE ALGEBRA

Graphing Non-Linear Functions – Part 1 (Graphing Quadratic Functions)

by

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NON-LINEAR FUNCTIONS

- These are functions that are not linear.
- Their graph is not a straight line.
- The degree of these functions is not 1.

- Non-linear functions can be:
- Quadratic functions – a polynomial of degree 2
- Cubic functions – a polynomial of degree 3
- Other higher order functions
- Exponential functions
- Logarithmic functions among others.

FOR THIS VIDEO,

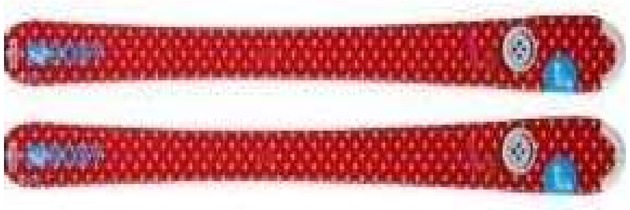
- We shall study the:
- Graphing of Quadratic Functions (**Vertical Parabolas**)

- The graph of a quadratic function is called a **parabola**

- Parabolas can be:
- **Vertical Parabolas** – graphs of quadratic functions of the form: $y = ax^2 + bx + c$ where $a \neq 0$
- Horizontal Parabolas – graphs of the quadratic functions of the form: $x = ay^2 + by + c$ where $a \neq 0$

SO, WHY STUDY PARABOLAS?

- Have you ever wondered why the light beam from the headlights of cars and from torches is so strong?
- Parabolas have a special reflecting property. Hence, the are used in the design automobile headlights, torch headlights, telescopes, television and radio antennae, among others.

- Why do the newest and most popular type of skis have parabolic cuts on both sides?
- Parabolic designs on skis will deform to a perfect arc, when under load. This shortens the turning area, and makes it much easier to turn the skis.

HAVE YOU ALSO NOTICED THAT:

- The shape of a water fountain is parabolic. It is a case of having the vertex as the greatest point on the parabola (in other words – maximum point).

- When you throw football or soccer or basketball, it bounces to the ground and bounces up, creating the shape of a parabola. In this case, the vertex is the lowest point on the parabola (in other words – minimum point).
- There are several more, but let’s move on.

Vocabulary Terms

- Quadratic Functions
- Parabolas
- Vertical Parabolas
- Vertex
- Axis
- Line of Symmetry
- Vertical Shifts
- Horizontal Shifts
- Domain
- Range

DEFINITIONS IN SIMPLE TERMS

- A quadratic function is a polynomial function of degree 2
- A parabola is the graph of a quadratic function
- A vertical parabola is the graph of a quadratic function of the form: $y = ax^2 + bx + c$ where $a \neq 0$
- The vertex of a vertical parabola is the *lowest point on the parabola* (in the case of a minimum point) or the *highest point on the parabola* (in the case of a maximum point).
- The axis of a vertical parabola is the vertical line through the vertex of the parabola.
- The line of symmetry of a vertical parabola is the axis in which if the parabola is folded across that axis, the two halves will be the same.

DEFINITIONS CONTINUED

- Vertical Shifts is a situation where we can graph a particular parabola by the translation or shifting of some units *up or down* of the parabola, $y = x^2$
- Horizontal Shifts is a situation where we can graph a particular parabola by the translation or shifting of some units *right or left* of the parabola, $y = x^2$
- The domain of a quadratic function is the set of values of the independent variable (x-values or input values) for which the function is defined.
- The range of a quadratic function is the set of values of the dependent variable (y-values or output values) for which the function is defined.

AS YOU CAN RECALL,

- $y = f(x)$ read as "*y is a function of x*"
- $y = ax^2 + bx + c$ where $a \neq 0$ – This is a quadratic function of x. It is called the general form of a quadratic function. This is also written as:
- $f(x) = ax^2 + bx + c$ where $a \neq 0$
- y is known as the dependent variable
- x is known as the independent variable

- **Bring it to "*Statistics*"**
- y is known as the response variable
- x is known as the predictor or explanatory variable

TO GRAPH PARABOLAS,

- We can either:
- Draw a Table of Values for some input values (x-values) , and determine their corresponding output values (y-values). Then, we can sketch our values on a graph using a suitable scale. In this case, it is important to consider negative, zero, and positive x-values. This is necessary to observe the behavior of the graph.
- Use a Graphing Calculator to graph the quadratic function directly. Some graphing calculators will sketch the graph only; while some will sketch the graph, as well as provide a table of values.

- For this presentation, we shall draw a *Table of Values*; and then use a *Graphing Calculator*.

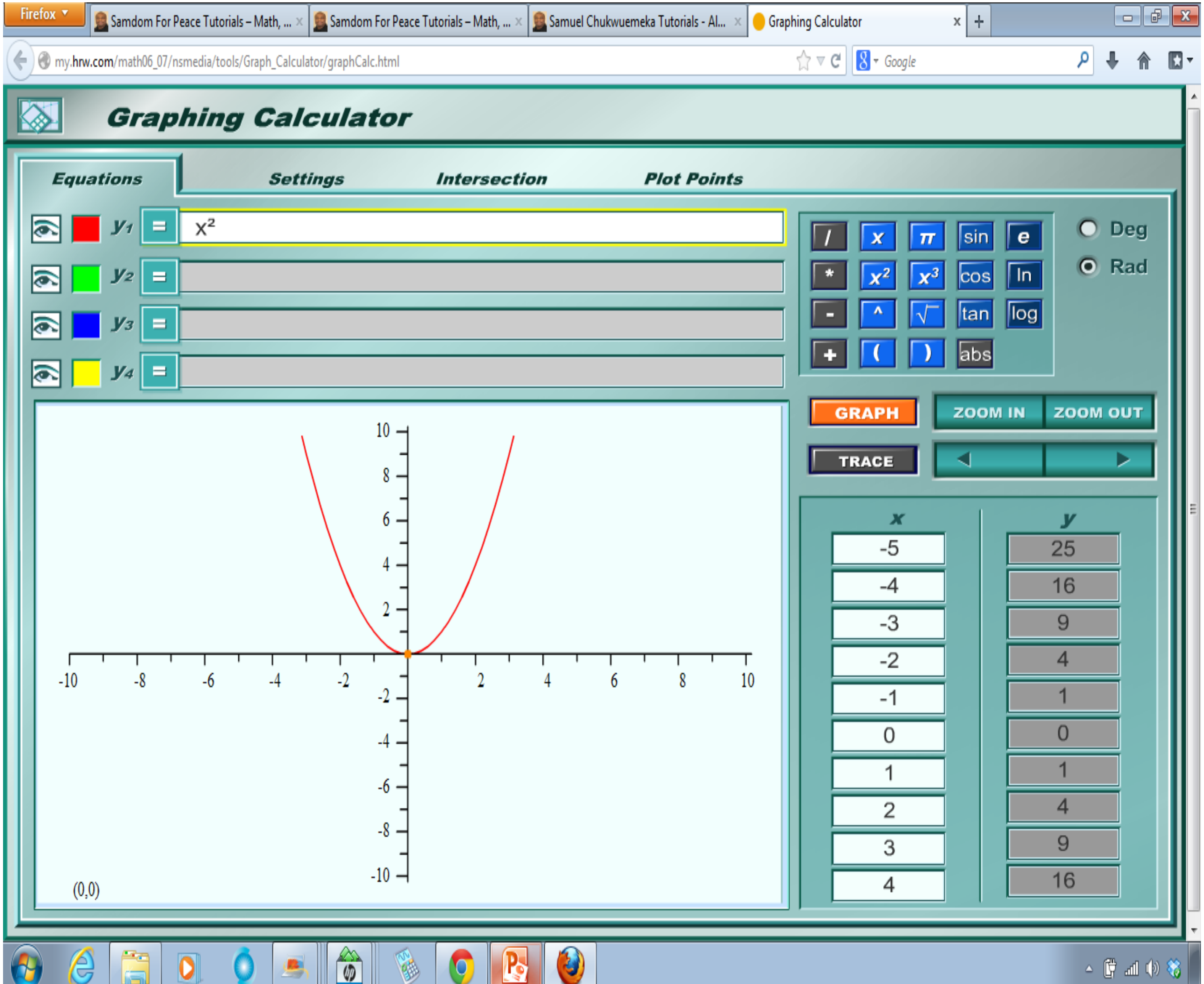
LET US BEGIN WITH THE GRAPH:

$y = x^2$

- Let us draw a Table of Values for $y = x^2$
- It is necessary to consider negative, zero, and positive values of x

x	y
-2	4
-1	1
0	0
1	1
2	4

- Then, let us use a graphing calculator to sketch the graph and study it. Use the graphing calculator on my website.


Graphing Calculator
my.hrw.com/math06_07/nsmedia/tools/Graph_Calculator/graphCalc.html
Equations
Settings
Intersection
Plot Points
y1 = x²
y2 =
y3 =
y4 =
sin
cos
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abs
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Rad
GRAPH
ZOOM IN
ZOOM OUT
TRACE
x
-5
-4
-3
-2
-1
0
1
2
3
4
y
25
16
9
4
1
0
1
4
9
16
(0,0)

WE NOTICE THAT THE:

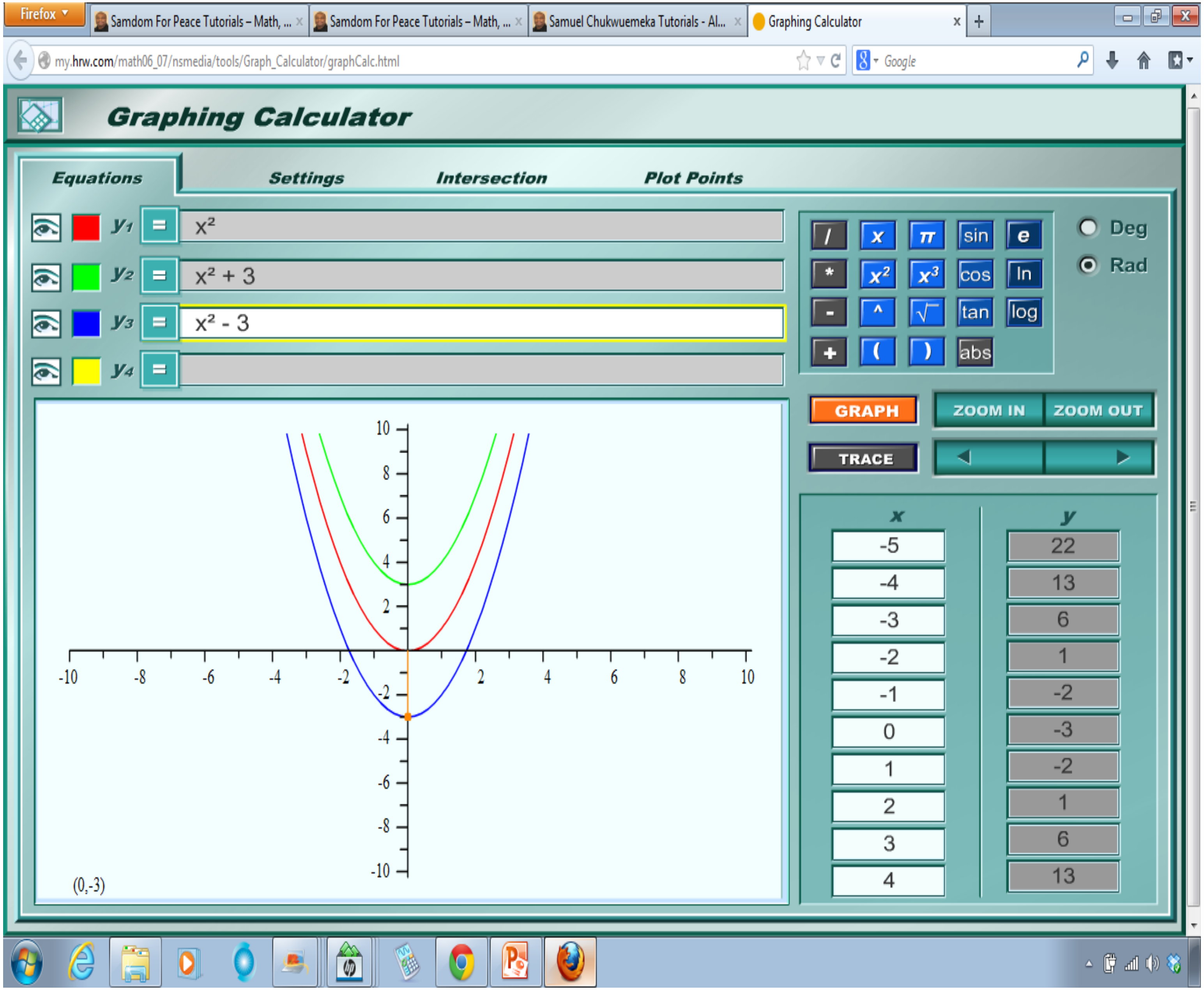
- Vertex: (0,0); opens up; minimum value
- Axis: $x = 0$
- Domain: $(-\infty, \infty)$ as x can be any real number
- Range: $[0, \infty)$ as y is always non-negative
- Let's deviate a bit: What is the difference between "non-negative" and "positive"?

- Let's now illustrate vertical shifts by graphing these parabolas:
- $y = x^2 + 3$
- $y = x^2 - 3$

BEGIN WITH A TABLE OF VALUES

$y = x^2 + 3$		$y = x^2 - 3$	
x	y	x	y
-2	7	-2	1
-1	4	-1	-2
0	3	0	-3
1	4	1	-2
2	7	2	1

$y = x^2 + 3$	$y = x^2 - 3$
Vertex: (0, 3)	Vertex: (0, -3)
Axis: x = 0	Axis: x = 0
Domain: (-∞, ∞)	Domain: (-∞, ∞)
Range: [3, ∞)	Range: [-3, ∞)



Graphing Calculator
my.hrw.com/math06_07/nsmedia/tools/Graph_Calculator/graphCalc.html
Equations
Settings
Intersection
Plot Points
y1 = x²
y2 = x² + 3
y3 = x² - 3
y4 =
Deg
Rad
GRAPH
ZOOM IN
ZOOM OUT
TRACE
(0,-3)
x
-5
-4
-3
-2
-1
0
1
2
3
4
y
22
13
6
1
-2
-3
-2
1
6
13

THIS MEANS THAT FOR:

- **Vertical Shifts**,

- The graph of $y = x^2 + m$ is a parabola
- The graph has the same shape as the graph of $y = x^2$
- The parabola is translated ***m*** units up if $m > 0$; and $|m|$ units down if $m < 0$
- The vertex of the parabola is $(0, m)$
- The axis of the parabola is: $x = 0$
- The domain of the parabola is $(-\infty, \infty)$
- The range of the parabola is $[m, \infty)$

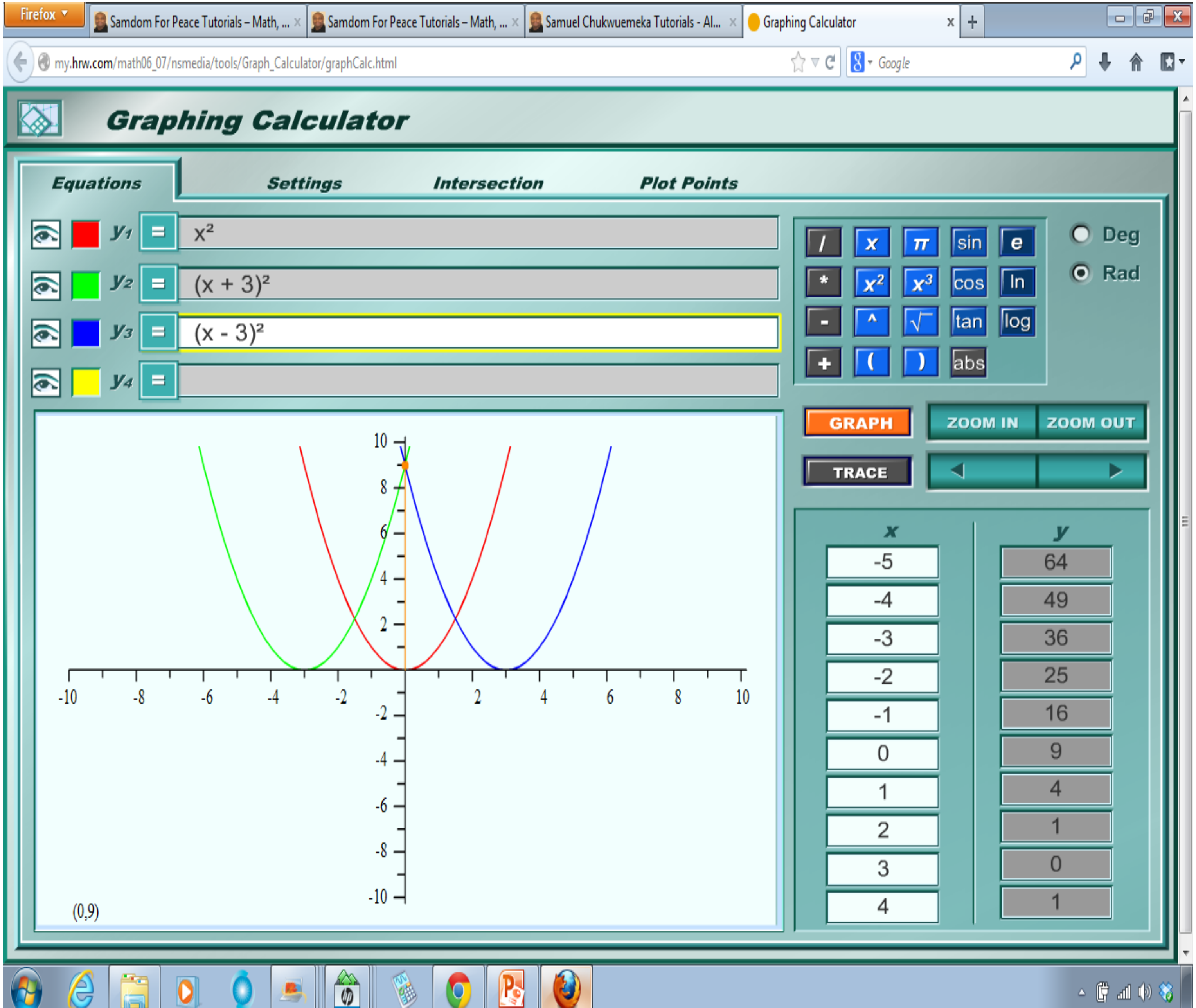
HORIZONTAL SHIFTS

- Let us illustrate horizontal shifts by graphing these parabolas:
- $y = (x + 3)^2$
- $y = (x – 3)^2$

BEGIN WITH A TABLE OF VALUES

$y = (x+3)^2$		$y = (x - 3)^2$	
x	y	x	y
-2	1	-2	25
-1	4	-1	16
0	9	0	9
1	16	1	4
2	25	2	1

$y = (x + 3)^2$	$y = (x - 3)^2$
Vertex: (-3, 0)	Vertex: (3, 0)
Axis: x = -3	Axis: x = 3
Domain: (-∞, ∞)	Domain: (-∞, ∞)
Range: [0, ∞)	Range: [0, ∞)


Graphing Calculator
Equations
Settings
Intersection
Plot Points
y1 = x²
y2 = (x + 3)²
y3 = (x - 3)²
y4 =
Deg
Rad
GRAPH
ZOOM IN
ZOOM OUT
TRACE
(0,9)
x
-5
-4
-3
-2
-1
0
1
2
3
4
y
64
49
36
25
16
9
4
1
0
1

THIS MEANS THAT FOR:

- **Horizontal Shifts**,

- The graph of $y = (x + n)^2$ is a parabola
- The graph has the same shape as the graph of $y = x^2$
- The parabola is translated ***n*** units to the left if $n > 0$; and $|n|$ units to the right if $n < 0$
- The vertex of the parabola is $(-n, 0)$
- The axis of the parabola is: $x = -n$
- The domain of the parabola is $(-\infty, \infty)$
- The range of the parabola is $[0, \infty)$

YOU CAN ALSO HAVE IT THIS WAY:

- **Horizontal Shifts**,

- The graph of $y = (x – n)^2$ is a parabola
- The graph has the same shape as the graph of $y = x^2$
- The parabola is translated ***n*** units to the right if $n > 0$; and $|n|$ units to the left if $n < 0$
- The vertex of the parabola is $(n, 0)$
- The axis of the parabola is: $x = n$
- The domain of the parabola is $(-\infty, \infty)$
- The range of the parabola is $[0, \infty)$

CAN YOU TELL THESE MOVEMENTS?

- From the graph of $y = x^2$;

- $y = (x + 3)^2 + 3$: Move the graph of x^2 3 units to the left, then 3 units up
- $y = (x + 3)^2 – 3$: Move the graph of x^2 3 units to the left, then 3 units down
- $y = (x – 3)^2 + 3$: Move the graph of x^2 3 units to the right, then 3 units up
- $y = (x – 3)^2 - 3$: Move the graph of x^2 3 units to the right, then 3 units down

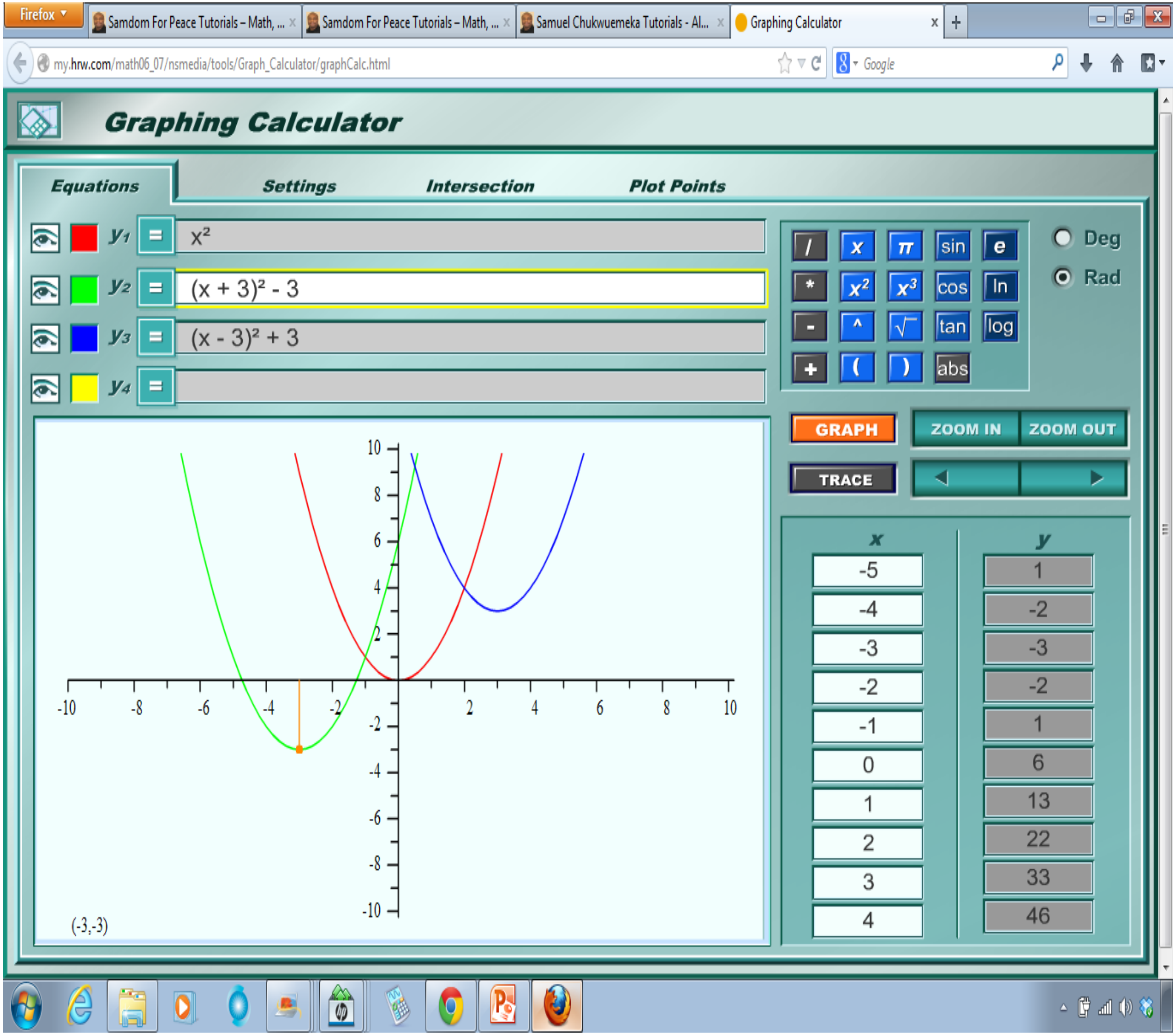
HORIZONTAL AND VERTICAL SHIFTS

- Let us illustrate horizontal and vertical shifts by graphing these parabolas:
- $y = (x + 3)^2 - 3$
- $y = (x – 3)^2 + 3$

BEGIN WITH A TABLE OF VALUES

$y = (x+3)^2 - 3$		$y = (x - 3)^2 + 3$	
x	y	x	y
-2	-2	-2	28
-1	1	-1	19
0	6	0	12
1	13	1	7
2	22	2	4

$y = (x + 3)^2 - 3$	$y = (x - 3)^2 + 3$
Vertex: (-3, -3)	Vertex: (3, 3)
Axis: x = -3	Axis: x = 3
Domain: (-∞, ∞)	Domain: (-∞, ∞)
Range: [-3, ∞)	Range: [3, ∞)


Graphing Calculator
Equations
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Intersection
Plot Points
y1 = x²
y2 = (x + 3)² - 3
y3 = (x - 3)² + 3
y4 =
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GRAPH
ZOOM IN
ZOOM OUT
TRACE
(-3,-3)
x
-5
-4
-3
-2
-1
0
1
2
3
4
y
1
-2
-3
-2
1
6
13
22
33
46

THIS MEANS THAT FOR:

- **Horizontal and Vertical Shifts**,

- The graph of $y = (x - n)^2 + m$ is a parabola
- The graph has the same shape as the graph of $y = x^2$
- The vertex of the parabola is (n, m)
- The axis of the parabola is the vertical line: $x = n$

CAN WE USE ANOTHER METHOD TO FIND THE VERTEX AND THE AXIS?

- We can use the "Completing the Square" method to find a formula for finding the vertex and axis of a vertical parabola (please view my video on "Completing the Square" method)
- For $y = ax^2 + bx + c$ where a = 0

- $The\ vertex\ is\ \left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$

and

- $The\ axis\ is\ the\ line: x = \frac{-b}{2a}$

LET'S LOOK AT A FORMER EXAMPLE

- Find the vertex and the axis of the parabola:
- $y = (x + 3)^2 – 3$
- Expanding the term gives:
- $y = (x + 3)(x + 3) – 3$
- $y = x^2 + 3x + 3x + 9 – 3$
- $y = x^2 + 6x + 6$. Compare to the form: $y = ax^2 + bx + c$
- This means that a = 1, b = 6, and c = 6
- $x = \frac{-b}{2a} = \frac{-6}{2*1} = \frac{-6}{2} = -3$
- For x = -3; $y = (-3)^2 + 6(-3) + 6$
- y = 9 – 18 + 6 = -3
- Therefore, the vertex is: (-3, -3)
- The axis is: x = -3

PREDICT THE SHAPE AND DIRECTION OF A VERTICAL PARABOLA

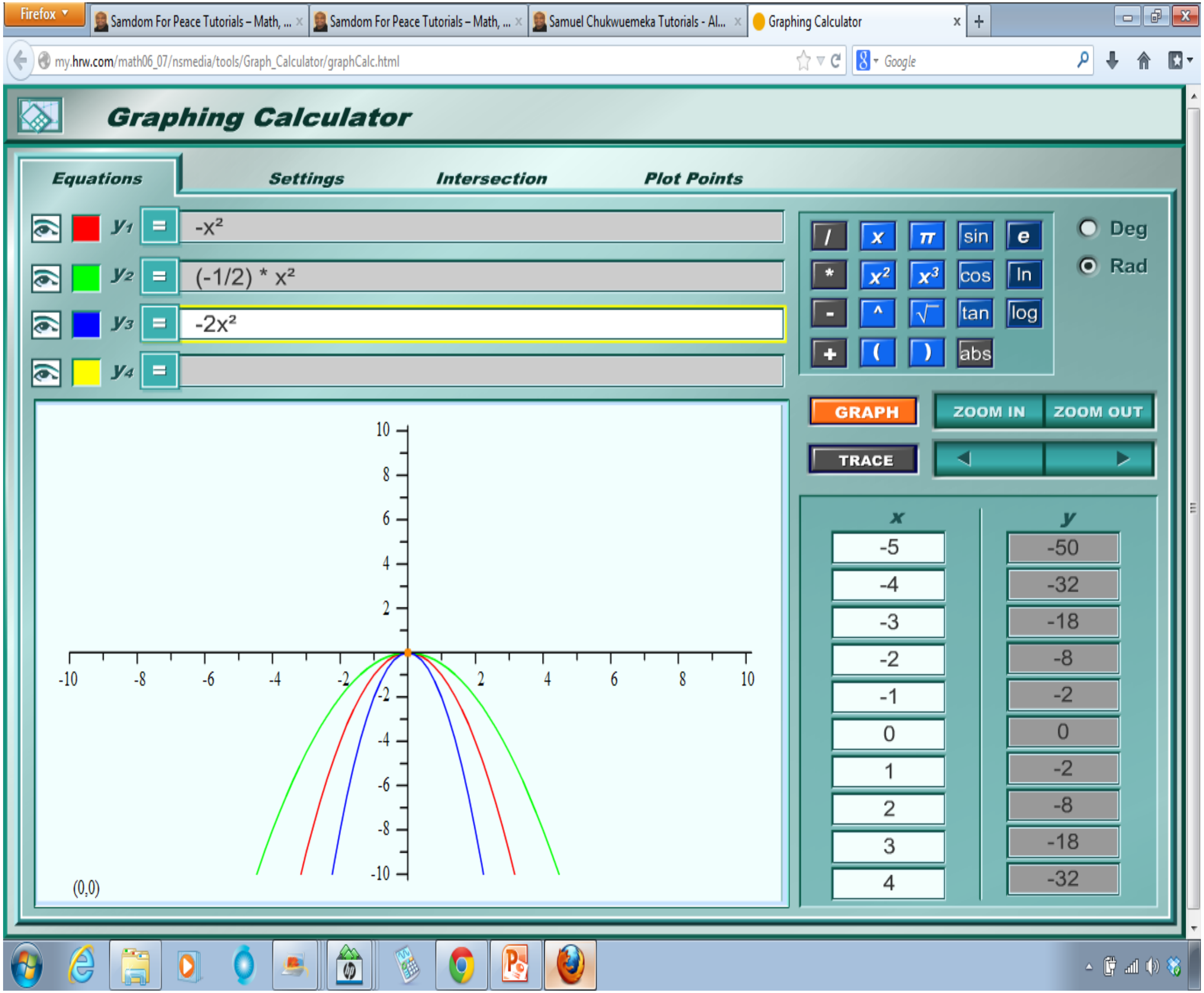
- It is important to note that:
- All parabolas do not open up
- All parabolas do not have the same shape as the graph of $y = x^2$
- We have been looking at parabolas where the coefficient of x^2 (which is "***a***") is positive. Do you think the graph may change if "***a***" was negative?
- Let us graph these parabolas:
- $y = -x^2$ (Here, a = -1)
- $y = -\frac{1}{2}x^2$ $(Here, a = -\frac{1}{2})$
- $y = -2x^2$ (Here, a = -2)

AS USUAL, LET'S BEGIN WITH A TABLE OF VALUES

$y = -x^2$		$y = -\frac{1}{2}x^2$		$y = -2x^2$	
x	y	x	y	x	y
-2	-4	-2	-2	-2	-8
-1	-1	-1	$-\frac{1}{2}$	-1	-2
0	0	0	0	0	0
1	-1	1	$-\frac{1}{2}$	1	-2
2	-4	2	-2	2	-8

WHAT DO WE NOTICE?

$y = -x^2$		$y = -\frac{1}{2}x^2$		$y = -2x^2$	
Vertex:	(0, 0)	Vertex:	(0, 0)	Vertex:	(0, 0)
Axis:	x = 0	Axis:	x = 0	Axis:	x = 0
Domain:	(-∞, ∞)	Domain:	(-∞, ∞)	Domain:	(-∞, ∞)
Range:	(-∞, 0]	Range:	(-∞, 0]	Range:	(-∞, 0]



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y1 = -x²
y2 = (-1/2) * x²
y3 = -2x²
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-8
3
-18
4
-32
10
8
6
4
2
-2
-4
-6
-8
-10
-10
-8
-6
-4
-2
2
4
6
8
10
(0,0)

DO YOU KNOW THAT:

- We shall notice the similar effect (but where the parabola opens up) with:
- $y = x^2; y = \frac{1}{2}x^2; and\ y = 2x^2$

- Do you want us to check it out?

SO, WE CAN SAY THAT:

- The graph of a parabola opens up if *a* is positive, and opens down if *a* is negative
- The graph is narrower than that of $y = x^2$ if $|a| > 1$
- The graph is narrower than that of $y = -x^2$ if $a < -1$
- The graph is wider than that of $y = x^2$ if $0 < |a| < 1$
- The graph is wider than that of $y = -x^2$ if $-1 < a < 0$

LET US NOW LOOK AT THE GRAPHING OF QUADRATIC FUNCTIONS IN GENERAL

- Recall that the general form of a quadratic function is:
- $y = ax^2 + bx + c$ where a = 0

- Sometimes, you shall be asked to graph a function that is that form (not the kind of ones we have been doing).

- What do you do?

THE STEPS ARE:

- Determine whether the graph opens up or down. (if $a > 0$, the parabola opens up; if $a < 0$, the parabola opens down; if $a = 0$, it is not a parabola. It is linear.)
- Find the vertex. You can use the vertex formula or the "Completing the Square" method
- Find the x- and y-intercepts. To find the x-intercept, put $y = 0$ and solve for x. to find the y-intercept, put $x = 0$ and solve for y. (You can use the discriminant to find the number of x-intercepts of a vertical parabola)
- Complete the graph by plotting the points. It is also necessary to find and plot additional points, using the symmetry about the axis.

USING THE DISCRIMINANT,

- Let us recall that the discriminant is:

$$b^2 – 4ac$$

- We can use the discriminant to find the number of x-intercepts of a vertical parabola

- If the discriminant is positive; then the parabola has two x-intercepts
- If the discriminant is zero; then the parabola has only one x-intercept
- If the discriminant is negative; then the parabola has no x-intercepts.

LET US DO AN EXAMPLE

- Graph the function: $x^2 + 7x + 10$
- Compare it to the general form: $ax^2 + bx + c$

a = 1; b = 7; c = 10

- 1st step: Since a > 0; the graph opens up
- 2nd step: Let us find the vertex using the vertex formula
- $x = -\frac{b}{2a} = -\frac{7}{2*1} = -\frac{7}{2} = -3.5$
- y = f(x) = f(-3.5) = $(-3.5)^2$ + 7(-3.5) + 10
- y = 12.25 – 24.5 + 10 = -2.25
- Vertex = (-3.5, -2.25)

LET US COMPLETE THE QUESTION

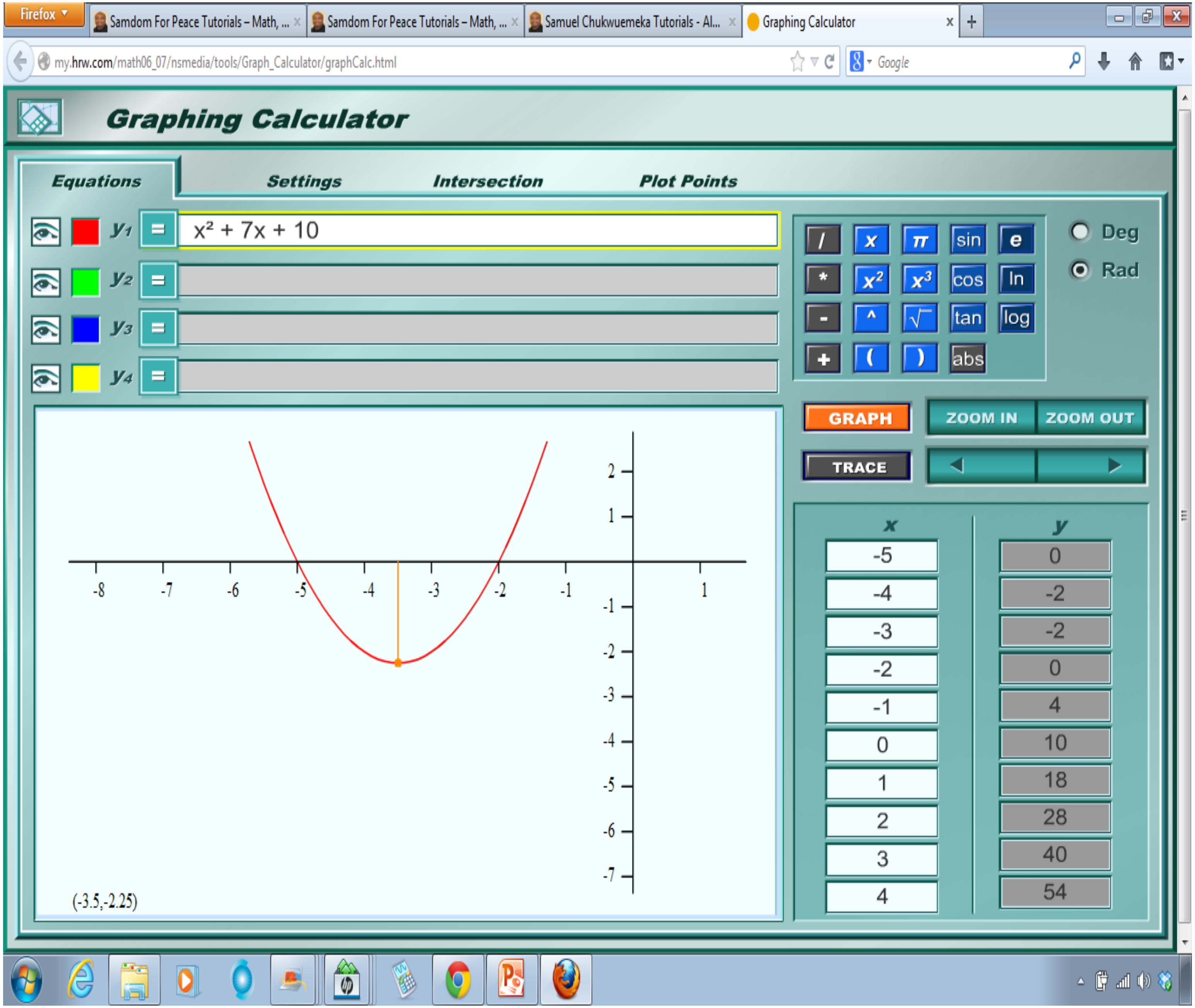
- 3rd step: the discriminant = $b^2 - 4ac$
- Discriminant = $7^2 - 4(1)(10) = 49 - 40 = 4$
- Since the discriminant is positive, we have two x-intercepts. Let us find them.
- Solve $x^2 + 7x + 10 = 0$
- Using the Factorization method (method of Factoring),
- We have that x = -5 or x = -2 (please view my video on the "Factoring")
- The x-intercepts are (-5, 0) and (-2, 0)
- $f(0) = 0^2 + 7(0) + 10 = 0 + 0 + 10 = 10$
- The y-intercept is: (0, 10)

WE CAN FIND ADDITIONAL POINTS

- By drawing a Table of Values:

x	y
-2	0
-1	4
0	10
1	18
2	28

- We can then sketch our graph!
- Let us see how this graph looks with a graphing calculator.
- Thank you for listening! Have a great day!!!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y3 =
y4 =
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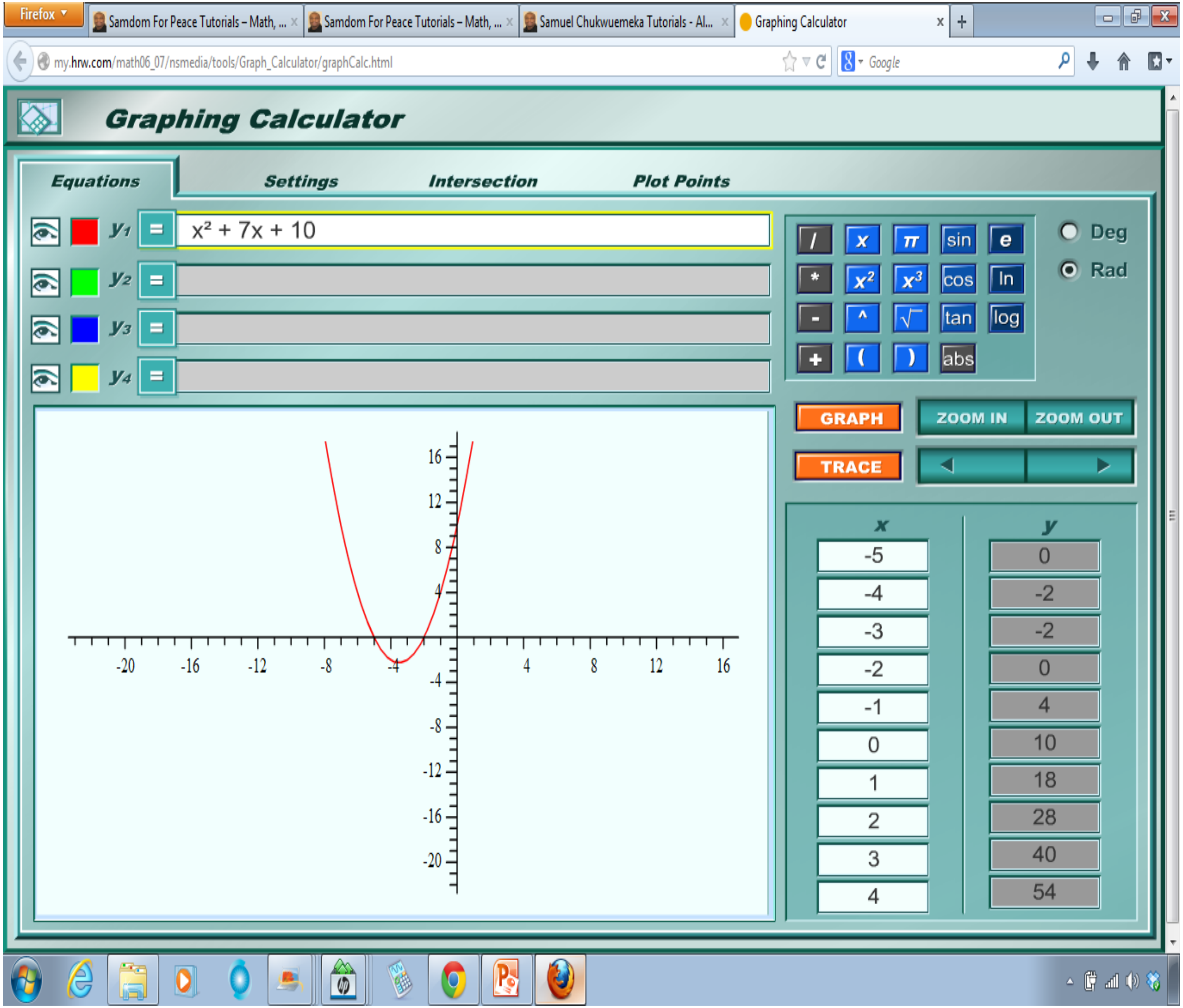
ZOOM IN
ZOOM OUT
TRACE
x: -5, -4, -3, -2, -1, 0, 1, 2, 3, 4
y: 0, -2, -2, 0, 4, 10, 18, 28, 40, 54
-8 -7 -6 -5 -4 -3 -2 -1 1
2 1 -1 -2 -3 -4 -5 -6 -7
(-3.5,-2.25)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y2 =
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y4 =
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GRAPH
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ZOOM IN
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-4
-3
-2
-1
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-2
-2
0
4
10
18
28
40
54