Implicit Differentiation

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Implicit Functions and Differentiation

□ In Calculus,

- **Y** is known as the *dependent* variable
- X is known as the independent variable
- □ Implicit functions are functions in which:
- y is expressed implicitly with x
- y is not expressed in terms of x; rather the function contains the sum, difference, product and/or quotient of both variables together
- □ Implicit Differentiation is:
- the differentiation of implicit functions
- the rate of change of y with respect to (wrt) changes in x of implicit functions

What do we need to know before we can differentiate implicitly?

Rules of Differentiation Power Rule: if y = axⁿ then dy/dx = naxⁿ⁻¹

Sum Rule: if y = u + vwhere y, u and v are functions of x then dy/dx = du/dx + dv/dx

Rules of Differentiation

Product Rule: if y = u * v

- Where u and v are functions of x,
- then $dy/dx = u^* (dv/dx) + v^* (du/dx)$

Function of a Function Rule or Chain Rule:

- If y is a function of u and u is a function of x,
- If y = f(u) and u = f(x),
- Then dy/dx = dy/du * du/dx

Rules of Differentiation

Quotient Rule: if y = u/v

- Where y, u and v are functions of x,
- Then dy/dx = (v * (du/dx) u * (dv/dx)) / v^2

What else do I need to know?

- Besides these rules, it is also important to know that:
- Differentiating:
- y wrt x \longrightarrow dy/dx
- y^2 wrt x \longrightarrow 2ydy/dx
- y^3 wrt x \longrightarrow $3y^2 dy/dx$

and so on.....

Can we do some examples?

- Differentiate implicitly $x^3y + y^3x = -7$
- <u>Solution</u>
- What rules do we need: **Power, Product and Sum Rules**
- Differentiating gives that
- $\checkmark 3x^{2}y + x^{3}dy/dx + y^{3} + 3xy^{2}dy/dx = 0$

$$\checkmark x^3 dy/dx + 3xy^2 dy/dx = 0 - 3x^2y - y^2$$

- ✓ $dy/dx (x^3 + 3xy^2) = -3x^2y y^3$
- $\checkmark dy/dx = -(3x^2y + y^3) / (x^3 + 3xy^2)$
- Please ask your questions.
- Ok. Go to the "Test" section and attempt the questions.